Aristotle University of Thessaloniki
School of Electrical & Computer Engineering
Photonics Group

http://photonics.ee.auth.gr

Single- and Multi-Channel Nonlinear Effects in Graphene-Enhanced Resonators

T. Christopoulos, O. Tsilipakos, G. Sinatkas, and **E. E. Kriezis**

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Motivation & Main Objectives

❑ Motivation

- ☑ Exploit **graphene's nonlinear properties** in practical **nanophotonic resonators**.
- ☑ Extend **perturbation theory** and **coupled mode theory** (CMT) frameworks for **2-D materials** (graphene, transition metal dichalcogenides, black phosphorus).
- ☑ Understand the **interplay of different nonlinear effects** using an accurate and efficient simulation framework.

❑ Main Objectives

- ☑ Develop a CMT framework for **single-** (Optical Bistability) and **multi-channel nonlinear effects** (Four-Wave Mixing, Saturable Absorption) for graphene-enhanced resonators in NIR and THz.
- ☑ **Validate** the CMT framework **against full-wave simulations**.
- ☑ Allow for the systematic **inclusion** of various **linear and nonlinear effects**, including carrier effects in Si.
- ☑ Demonstrate that nonlinear optical effects are **exploitable** at **practical power levels** by proper engineering.

Presentation Overview / At a glance

Optical properties of graphene: quick review

Linear surface conductivity
Nonlinear surface conductivity tensor

Optical Bistability

Bistability with TW resonators in the NIR
Bistability with TW resonators in the FIR

Degenerate Four Wave Mixing

FWM with SW resonators in the FIR

Saturable Absorption

Intensity-dependent conductivity model
SA-induced switching/routing in nanophotonic disk resonators

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Optical properties of graphene / Graphene as a conducting sheet

Maxwell's curl equations, including both linear and nonlinear contributions (frequency-domain). 2D materials are introduced as a surface current density:

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu_0\tilde{\mathbf{H}}$$
$$\nabla \times \tilde{\mathbf{H}} = +j\omega\varepsilon_0\tilde{\mathbf{E}} + j\omega\tilde{\mathbf{P}}_{\mathrm{lin}} + \tilde{\mathbf{J}}_{\mathrm{lin}}\delta_s(\mathbf{r}) + j\omega\tilde{\mathbf{P}}_{\mathrm{NL}} + \tilde{\mathbf{J}}_{\mathrm{NL}}\delta_s(\mathbf{r})$$

perturbation terms

$$\tilde{\mathbf{P}}_{\mathrm{NL}} << \tilde{\mathbf{P}}_{\mathrm{lin}} = \varepsilon_0\bar{\chi}^{(1)}\tilde{\mathbf{E}}$$
$$\tilde{\mathbf{J}}_{\mathrm{NL}} << \tilde{\mathbf{J}}_{\mathrm{lin}} = \bar{\sigma}_s^{(1)}\tilde{\mathbf{E}}$$

Surface current density on graphene (time-domain):

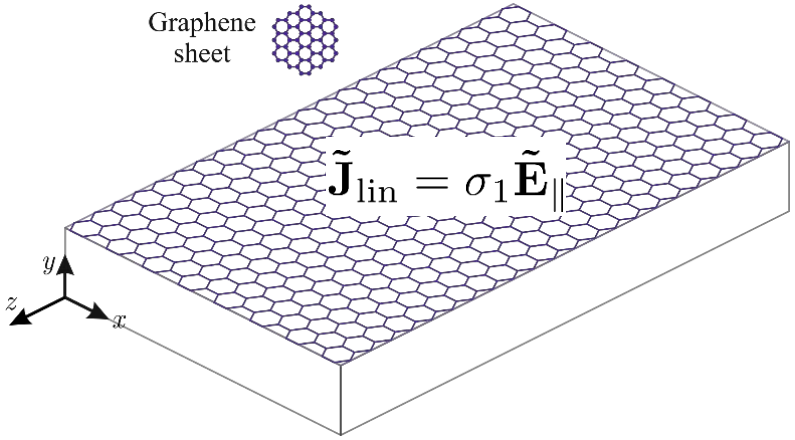
$$\mathcal{J}_s = \bar{\sigma}^{(1)} \mid \mathcal{E} + \bar{\sigma}^{(2)} \mid \mathcal{E}\mathcal{E} + \bar{\sigma}^{(3)} \mid \mathcal{E}\mathcal{E}\mathcal{E} + \ldots$$

important important

$\bar{\sigma}_s^{(n)}$ Surface conductivity tensor, rank $(n+1)$, in $[\mathrm{S}(\mathrm{m}/\mathrm{V})^{n\text{-}1}]$

Overall current density

$$\mathbf{J} = \mathbf{J}_s\delta_s(\mathbf{r}) = (\mathbf{J}_{\mathrm{lin}} + \mathbf{J}_{\mathrm{NL}})\delta_s(\mathbf{r})$$

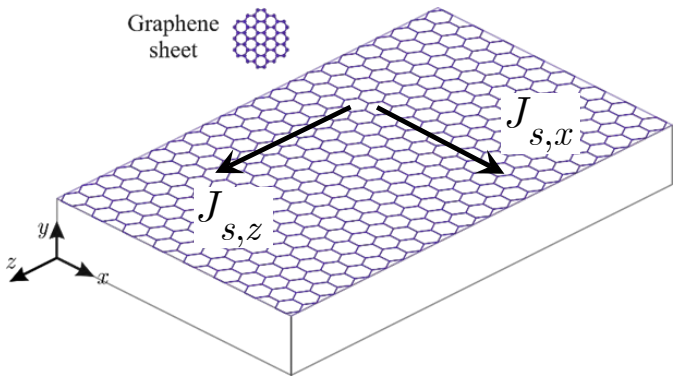


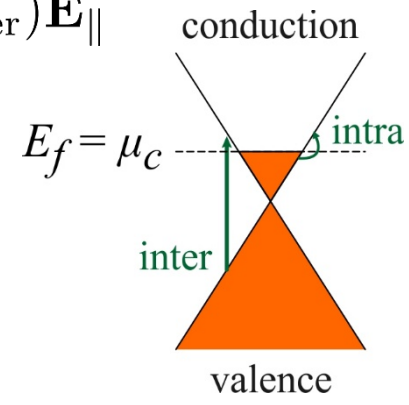
$$\bar{\sigma}_s^{(1)} = \begin{bmatrix} \sigma_{s,xx} & 0 & \sigma_{s,xz} \\ 0 & 0 & 0 \\ \sigma_{s,zx} & 0 & \sigma_{s,zz} \end{bmatrix}$$

$$\sigma_{s,xx} = \sigma_{s,zz} \equiv \sigma_1, \ \sigma_{s,xz} = \sigma_{s,zx} \simeq 0$$

Optical properties of graphene / Linear surface conductivity in the NIR

$$\mathbf{J}_s = \mathbf{J}_{s,\text{intra}} + \mathbf{J}_{s,\text{inter}} = (\sigma_{\text{intra}} + \sigma_{\text{inter}})\mathbf{E}_{\parallel}$$





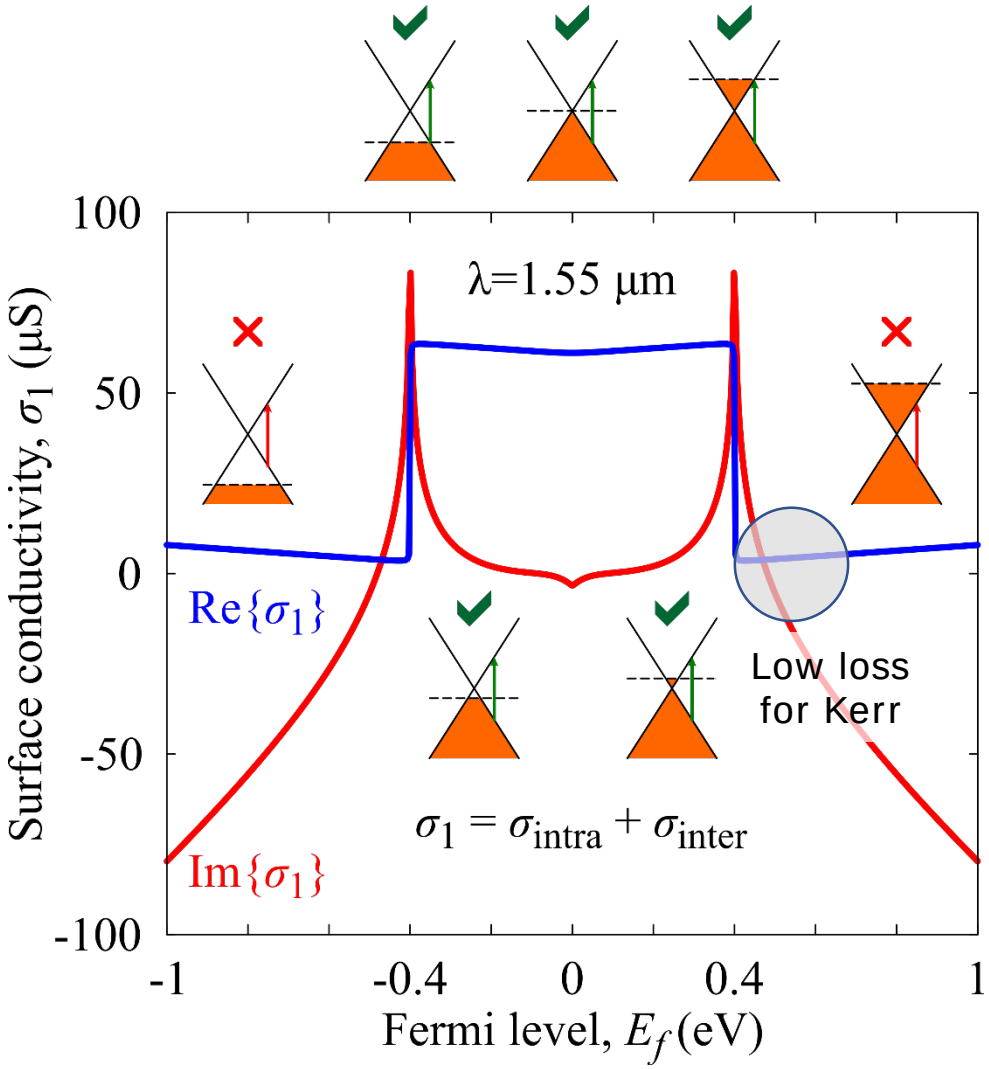
- **Symmetric** change around Dirac point ($\mu_c = 0$)
- **Step behavior** for $\mathrm{Re}\{\sigma\}$ at $E_f = \pm\, 0.4\ \mathrm{eV}$ for $\lambda = 1.55\ \mu\mathrm{m}$

$$\sigma_1 = \sigma_{\text{intra}} + \sigma_{\text{inter}}$$

$$\sigma_{\text{intra}}(\mu_c) = \frac{-je^2 k_B T}{\pi\hbar^2(\omega - j/\tau_1)}\left[\frac{\mu_c}{k_B T} + 2\ln(e^{-\mu_c/k_B T} + 1)\right]$$

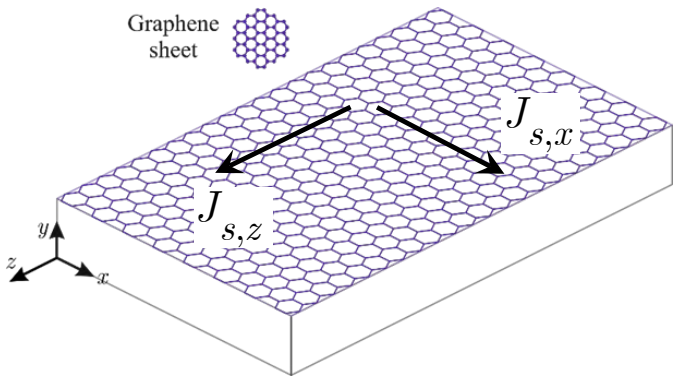
$$\sigma_{\text{inter}}(\mu_c) = \frac{-je^2}{4\pi\hbar}\ln\left[\frac{2|\mu_c| - \hbar(\omega - j/\tau_2)}{2|\mu_c| + \hbar(\omega - j/\tau_2)}\right]$$

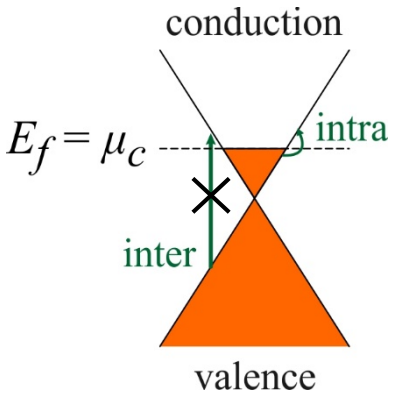
Hanson, *J. Appl. Phys.* **103**, 064302, 2008



Optical properties of graphene / Linear surface conductivity in the FIR

$$\mathbf{J}_s = \mathbf{J}_{s,\text{intra}} = \sigma_{\text{intra}}\mathbf{E}_{||}$$



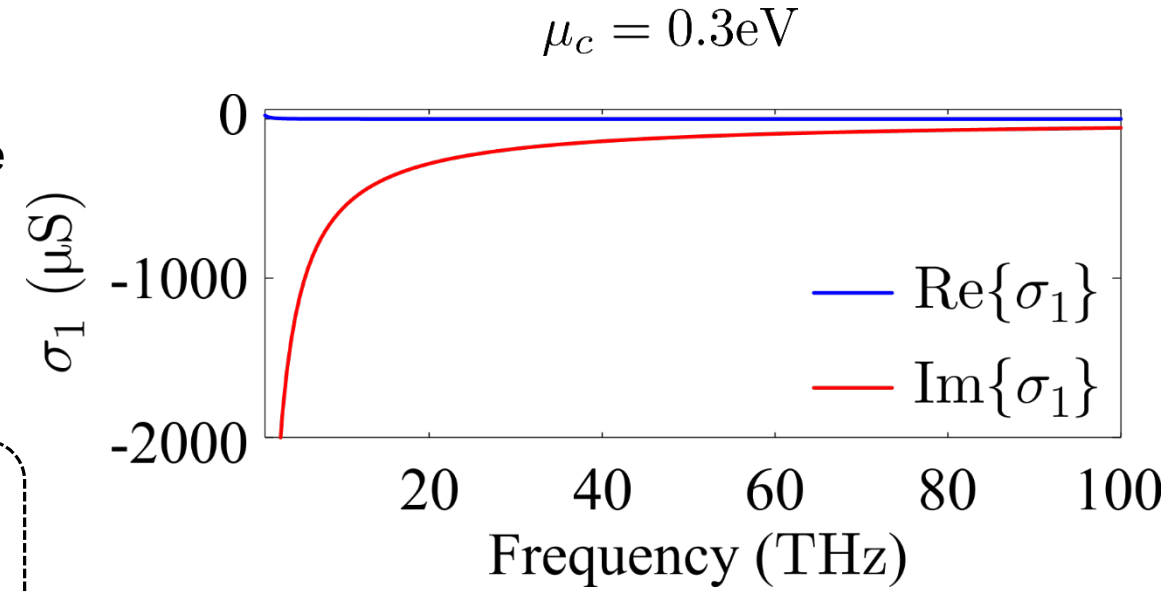


- **Only intraband** transitions allowed: Drude-like response
- **Low losses** (small $\mathrm{Re}\{\sigma\}$) and **plasmonic response** (high $\mathrm{Im}\{\sigma\}$)
- **Strong dispersion**

$$\sigma_1 = \sigma_{\text{intra}}$$

$$\sigma_{\text{intra}}(\mu_c) = \frac{-je^2 k_B T}{\pi\hbar^2(\omega - j/\tau_1)}\left[\frac{\mu_c}{k_B T} + 2\ln(e^{-\mu_c/k_B T} + 1)\right]$$

Falkovsky, *Phys. Usp.* **51**, 887, 2008



Optical properties of graphene / Nonlinear surface conductivity tensor

Third-order nonlinear term, which accounts for the **Kerr** effect, **TPA**, **THG** and **FWM**:

$$J_{\mathrm{NL},j}(\omega_0) = \mathbf{J}_{\mathrm{NL}} \cdot \hat{\mathbf{j}} = \frac{3}{4} \sum_{klm}^{xyz} \sigma^{(3)}_{s,jklm}(\omega_0; \omega_0, -\omega_0, \omega_0) E_k E_l^* E_m$$

j-th Cartesian component of the nonlinear surface current - refers to monochromatic or quasi-monochromatic waves (SVE)

$\bar{\sigma}_s^{(3)}$ Surface conductivity tensor, 4th-rank (81 elements), units [S(m/V)²]

- Hexagonal symmetry group
- Normal to sheet component of J_{NL} vanishes

Source	Non-zero	Independent
Hexagonal 2D crystal	8+6	3+3
Gorbach et al., OL, 2013	8+6	1+1
Cheng et al., New J. Phys, 2014	8	2
Simplest model	8	1

Up to **14 non-zero** elements & up to **6** are **independent**!

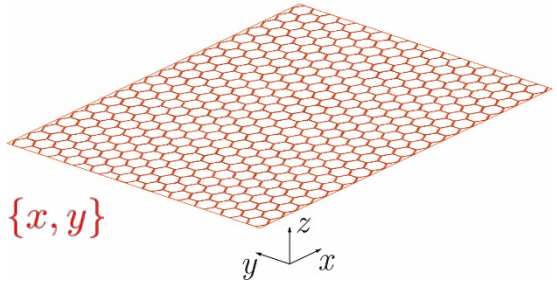
Simplest form (8 non-zero, 1 independent), i.e. 2D-equivalent of an isotropic bulk (3D) medium:

$$\sigma^{(3)}_{s,jklm} = \sigma_3 \frac{1}{3} \left(\delta_{jk}\delta_{lm} + \delta_{jm}\delta_{kl} + \delta_{jl}\delta_{mk} \right)$$

For example, a graphene sheet normal to the z-axis

$$\sigma^{(3)}_{s,xxxx} = \sigma^{(3)}_{s,yyyy} = \sigma_3$$

$$\sigma^{(3)}_{s,mmnn} = \sigma^{(3)}_{s,mnmn} = \sigma^{(3)}_{s,mnnm} = \frac{\sigma_3}{3} \quad \{m,n\} = \{x,y\}$$



Optical properties of graphene / Nonlinear surface conductivity tensor

$$\sigma^{(3)}_{s,jklm} = \sigma_3 \frac{1}{3}\left(\delta_{jk}\delta_{lm} + \delta_{jm}\delta_{kl} + \delta_{jl}\delta_{mk}\right)$$

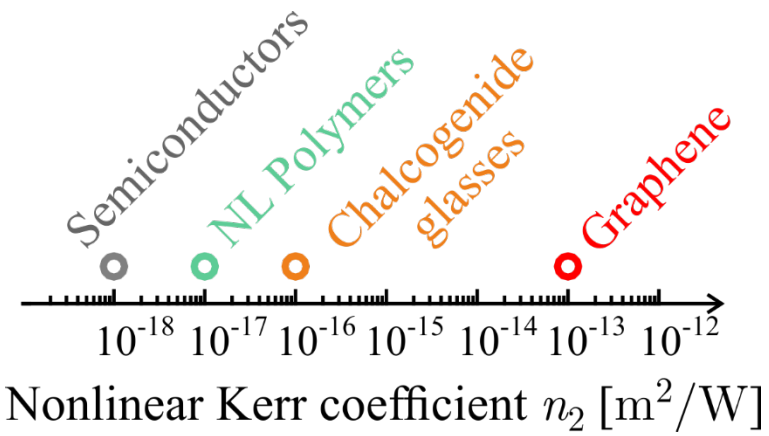
Nonlinear surface current solely depends on the **tangential** E-field

$$\mathbf{J}_{\mathrm{NL}} = \frac{1}{4}\sigma_3\left[2(\mathbf{E}_{0,\parallel}\cdot\mathbf{E}^*_{0,\parallel})\mathbf{E}_{0,\parallel} + (\mathbf{E}_{0,\parallel}\cdot\mathbf{E}_{0,\parallel})\mathbf{E}^*_{0,\parallel}\right]$$

NIR (1550 nm)	Equivalent nonlinear index n_2 [m²/W]	Surface conductivity σ_3 [S(m/V)²]
Demeteriou, *Opt. Express* **24**, 13033, 2016	-0.8×10^{-13}	$-j1.3 \times 10^{-21}$
Dremetsika, *Opt. Lett.* **41**, 3281, 2016	-1.1×10^{-13}	$-j1.7 \times 10^{-21}$
Vermeulen, *Phys. Rev. Applied* **6**, 044006, 2016	-1.0×10^{-13}	$-j1.6 \times 10^{-21}$
Alexander, *ACS Photonics* **5**, 4944, 2018	-	(up to) $-j2.0 \times 10^{-19}$
Alexander, *ACS Photonics* **4**, 3039, 2017	-	(up to) $-j4.3 \times 10^{-19}$

FIR (10 THz)	Equivalent nonlinear index n_2 [m²/W]	Surface conductivity σ_3 [S(m/V)²]
Mikhailov, *J. Phys.-Condes. Matter* **20**, 384204, 2008	$+2.4 \times 10^{-13}$	$+j4.7 \times 10^{-19}$

- Graphene behaves as a **self-defocusing material** in the **NIR**.
- **High nonlinearity** but the **overlap** with the optical mode may be **weak**.
- Graphene behaves as **self-focusing material** in the **FIR**.



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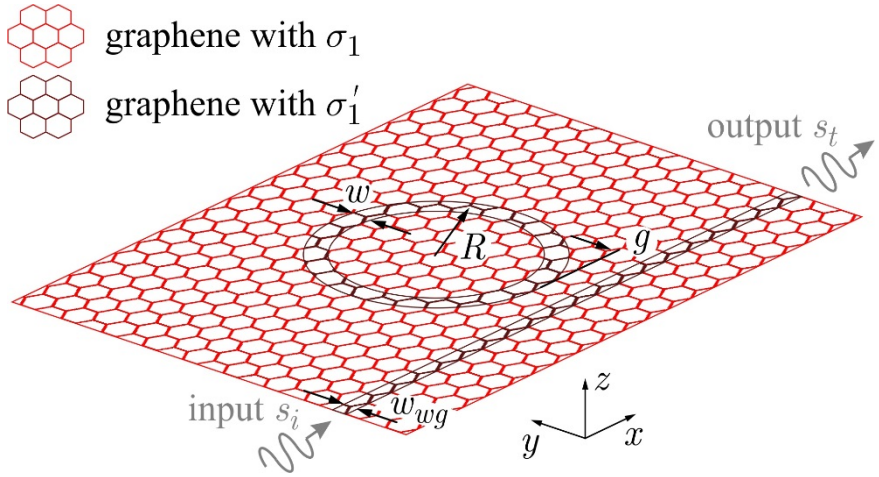
Optical Bistability / Basic concepts

Hysteresis Loop (Bistability): develops in systems exhibiting **nonlinearity** and **feedback**

- **Travelling-Wave resonator** → provides the feedback
- **Intensity buildup** inside the resonator → Manifestation of nonlinearities

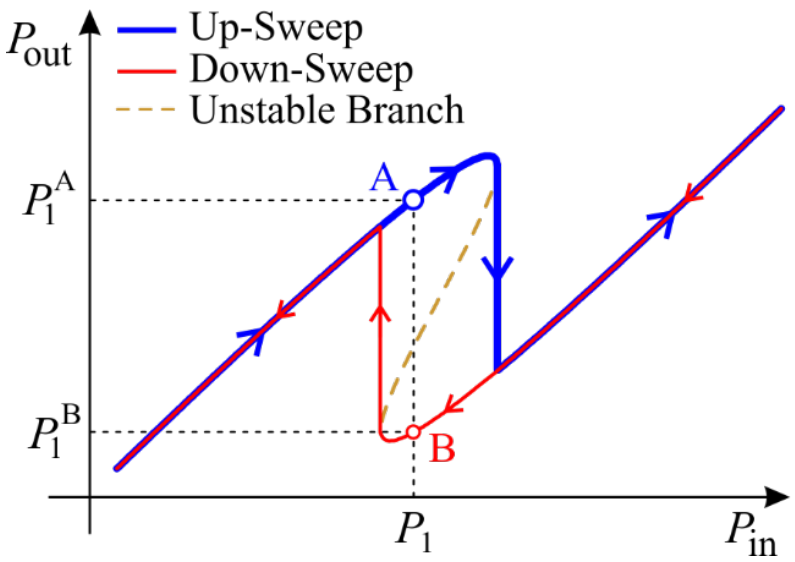


$$\sigma^{(3)} \rightarrow \Delta\sigma \propto |\mathbf{E}|^2 \rightarrow \Delta\omega_{\text{res}}$$

Perturbation Theory

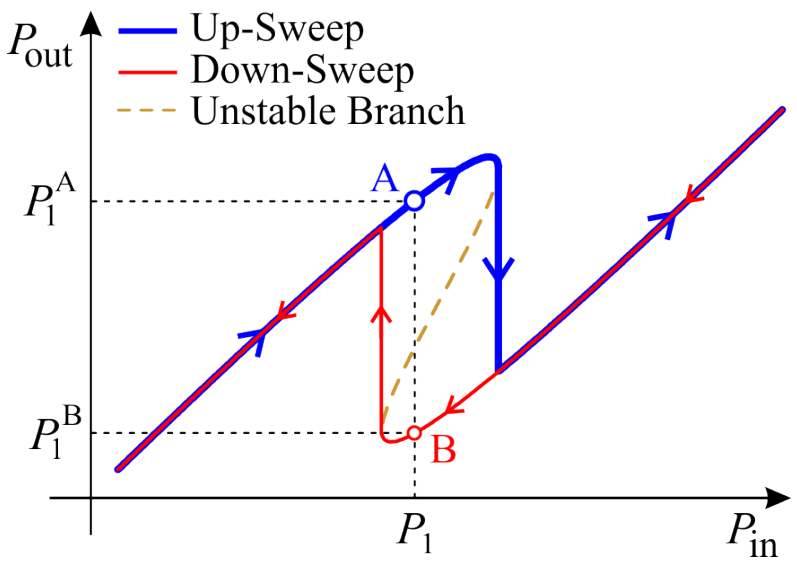
$$\Delta\omega = -\gamma\, W_{\text{res}}$$

- $\Delta\omega < 0$
- Proportional to energy *W*
- Estimated from linear calculations

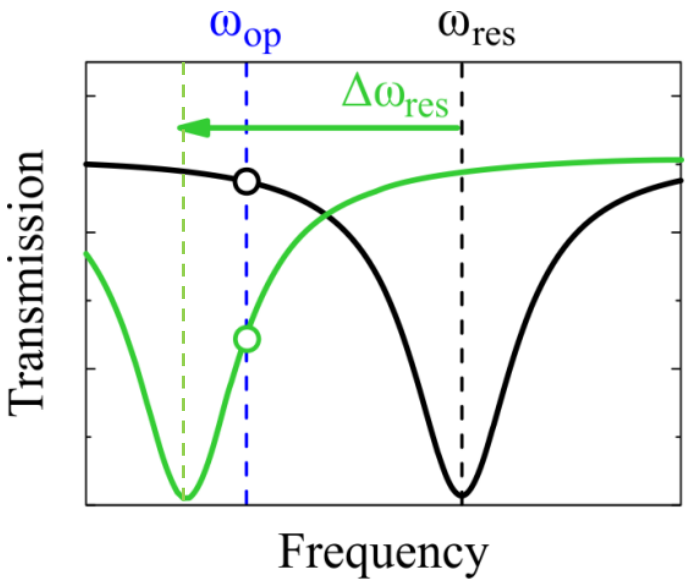


Optical Bistability / Basic concepts

Nonlinearity + optical feedback
→ Hysteresis loop (bistability)

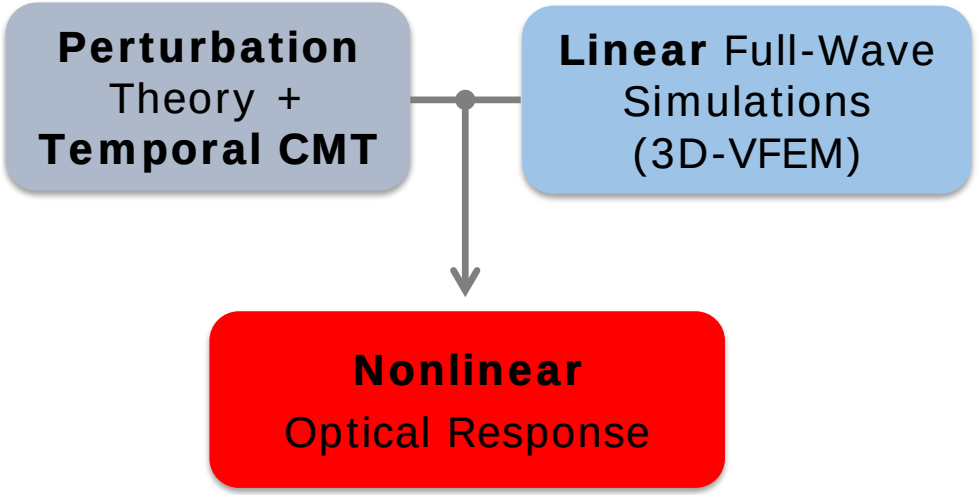


Prerequisite:
$\omega_{op} < \omega_{res}$ if $\Delta\omega_{res} < 0$



Increase in transmission

Mathematical Framework



Bravo-Abad, *J. Lightwave Technol.*, **25**, 2539, 2007

Optical Bistability / Perturbation theory in graphene resonators

Nonlinear frequency shift with both polarization & current density nonlinearities

Bulk nonlinear media — Includes the 2D sheet current

$$\frac{\Delta\omega}{\omega_0} = -\frac{\int_V \mathbf{P_{NL}} \cdot \mathbf{E_0^*} \mathrm{d}V - j\frac{1}{\omega_0}\int_V \mathbf{J_{NL}} \cdot \mathbf{E_0^*} \mathrm{d}V}{\int_V \frac{\partial\left\{\omega\varepsilon_0\overline{\varepsilon}_r + \overline{\sigma}_{\mathrm{Im}}^{(1)}\right\}}{\partial\omega}\mathbf{E_0} \cdot \mathbf{E_0^*} \mathrm{d}V + \mu_0 \int_V \mathbf{H_0} \cdot \mathbf{H_0^*} \mathrm{d}V}$$

Dispersive electric energy & complex conductivity contribution

Christopoulos, *Phys. Rev. E*, **94**, 062219, 2016

Complex nonlinear frequency shift, accounting both for the Kerr effect and TPA

Total stored energy

$$\Delta\omega = \left[-\gamma^{\mathrm{Kerr}} + j\gamma^{\mathrm{TPA}}\right] W_{\mathrm{res}} = \left[-\left(\gamma_b^{\mathrm{Kerr}} + \gamma_s^{\mathrm{Kerr}}\right) + j\left(\gamma_b^{\mathrm{TPA}} + \gamma_s^{\mathrm{TPA}}\right)\right] W_{\mathrm{res}}$$

$$\gamma_b^{\mathrm{Kerr}} = 4\left(\frac{\omega_0}{c_0}\right)^3 \omega_0 c_0 \kappa_b^{\mathrm{Kerr}} n_2^{\max}$$

Bulk nonlinear feedback parameter

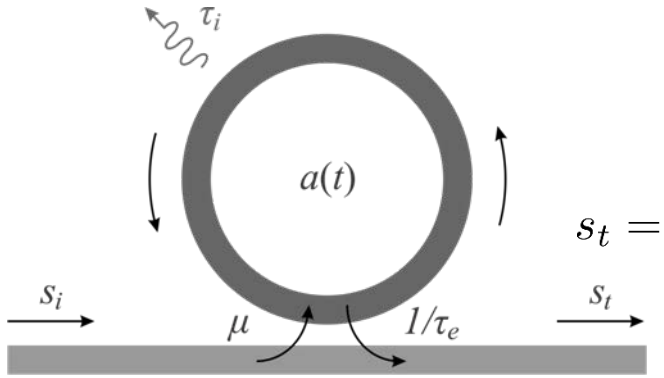
$$\kappa_b^{\mathrm{Kerr}} = \left(\frac{c_0}{\omega_0}\right)^3 \frac{\frac{1}{3}\int_V n_2 n^2 \left(2|\mathbf{E_0}|^4 + |\mathbf{E_0} \cdot \mathbf{E_0}|^2\right) \mathrm{d}V}{\left[\int_V \frac{\partial\{\omega\varepsilon_r\}}{\partial\omega}|\mathbf{E_0}|^2 \mathrm{d}V + \eta_0^2 \int_V |\mathbf{H_0}|^2 \mathrm{d}V + \frac{1}{\varepsilon_0}\int_S \frac{\partial\sigma_{1,\mathrm{Im}}}{\partial\omega}|\mathbf{E_{0,\parallel}}|^2 \mathrm{d}S\right]^2 n_2^{\max}}$$

$$\gamma_s^{\mathrm{Kerr}} = \left(\frac{\omega_0}{c_0}\right)^4 \kappa_s^{\mathrm{Kerr}} \frac{\sigma_{3,\mathrm{Im}}^{\max}}{\varepsilon_0^2}$$

Surface nonlinear feedback parameter

$$\kappa_s^{\mathrm{Kerr}} = \left(\frac{c_0}{\omega_0}\right)^4 \frac{\int_S \sigma_{3,\mathrm{Im}} \left(2|\mathbf{E_{0,\parallel}}|^4 + |\mathbf{E_{0,\parallel}} \cdot \mathbf{E_{0,\parallel}}|^2\right) \mathrm{d}S}{\left[\int_V \frac{\partial\{\omega\varepsilon_r\}}{\partial\omega}|\mathbf{E_0}|^2 \mathrm{d}V + \eta_0^2 \int_V |\mathbf{H_0}|^2 \mathrm{d}V + \frac{1}{\varepsilon_0}\int_S \frac{\partial\sigma_{1,\mathrm{Im}}}{\partial\omega}|\mathbf{E_{0,\parallel}}|^2 \mathrm{d}S\right]^2 \sigma_{3,\mathrm{Im}}^{\max}}$$

Optical Bistability / Coupled Mode Theory (CMT) in time / TW resonators



$$s_t = s_i + j\sqrt{\frac{2}{\tau_e}}a$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = j\left(\omega_0 - \gamma^{\mathrm{Kerr}}|a|^2\right)a - \left(\frac{1}{\tau_i} + \frac{1}{\tau_e} + \gamma^{\mathrm{TPA}}|a|^2\right)a + j\sqrt{\frac{2}{\tau_e}}s_i$$

	Quantity	Expression
$a(t)$	cavity amplitude	$W_{\mathrm{res}} = \lvert a(t)\rvert^2$
ω_0	unperturbed resonance frequency	
$\Delta\omega$	nonlinear frequency shift	$(-\gamma^{\mathrm{Kerr}} + j\gamma^{\mathrm{TPA}})\lvert a\rvert^2$
τ	photon lifetime	$\tau = 2Q/\omega_0$
μ	coupling coefficient	$\mu = j\sqrt{2/\tau_e}$
$s(t)$	waveguide mode amplitude	$\lvert s\rvert^2 \equiv P$

Steady-state (CW) response

System of polynomial equations

$$\frac{p_{\mathrm{out}}}{p_{\mathrm{in}}} = \frac{[\delta + \mathrm{sgn}(\gamma^{\mathrm{Kerr}})p_i]^2 + (1 - r_Q + r_{\mathrm{TPA}}p_i)^2}{[\delta + \mathrm{sgn}(\gamma^{\mathrm{Kerr}})p_i]^2 + (1 + r_Q + r_{\mathrm{TPA}}p_i)^2}$$

$$p_{\mathrm{TPA}} = r_{\mathrm{TPA}}p_i^2, p_i = p_{\mathrm{in}} - p_{\mathrm{out}} - p_{\mathrm{TPA}}$$

Tsilipakos, *J. Lightwave Technol.*, **34**, 1333, 2016

$\delta = \tau_i(\omega - \omega_0)$ Normalized detuning

$r_Q = Q_i/Q_e$ Q-factor ratio

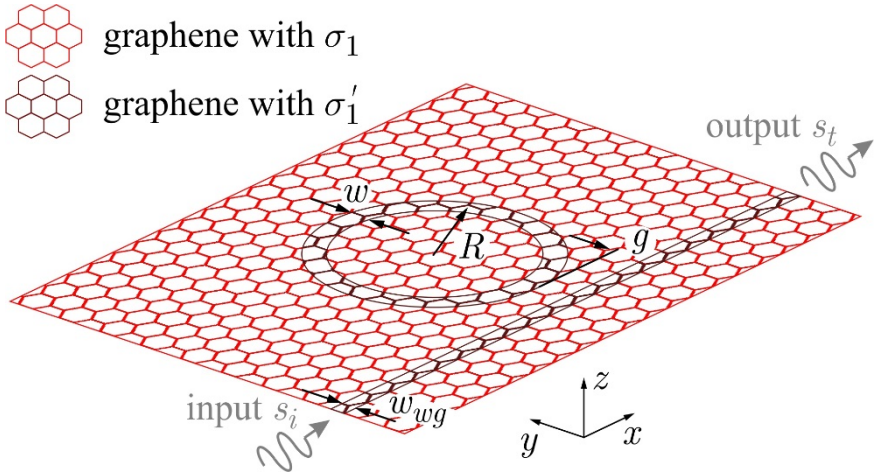
$P_0 = \dfrac{2}{\tau_i^2|\gamma^{\mathrm{Kerr}}|}$ Characteristic power

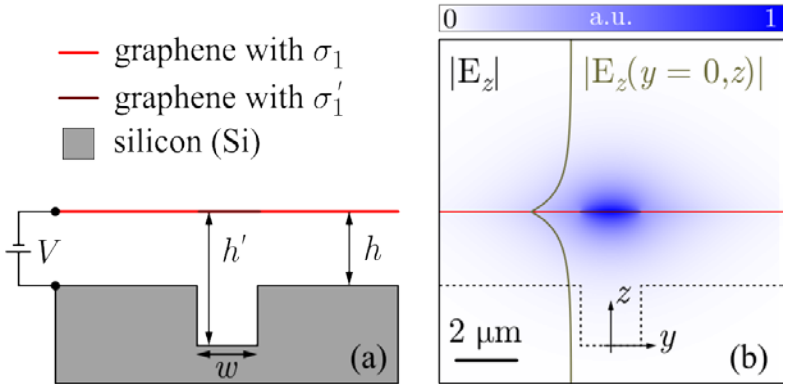
❖ For appropriate input power & detuning **three real positive solutions → bistability**

$$\delta < -\delta_{\mathrm{th}} \quad \text{or} \quad \delta > \delta_{\mathrm{th}}$$

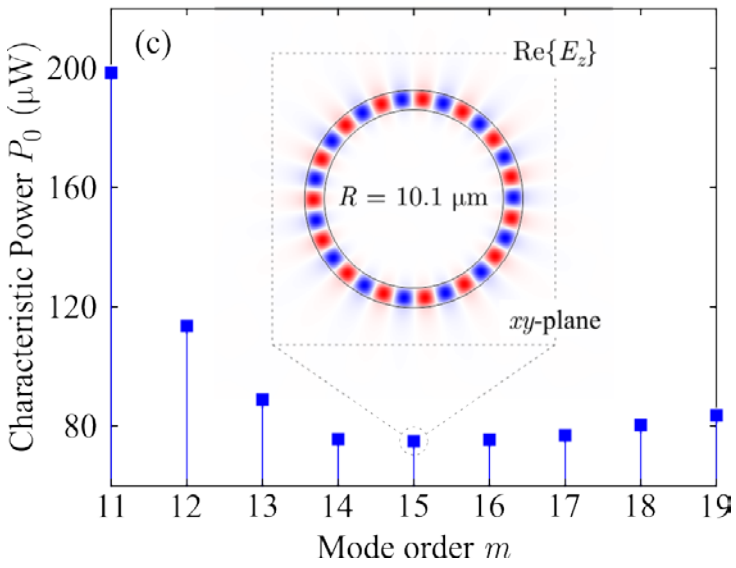
$$\delta_{\mathrm{th}} = \frac{(1 + r_Q)(\sqrt{3} + r_{\mathrm{TPA}})}{1 - \sqrt{3}\, r_{\mathrm{TPA}}}$$ Detuning threshold

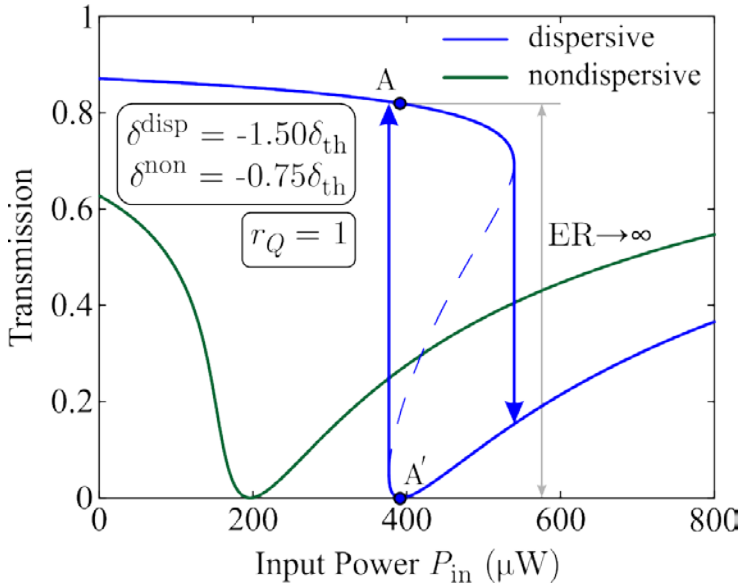
Optical Bistability / Graphene Nanoribbon (GNR) TW resonator in FIR frequencies





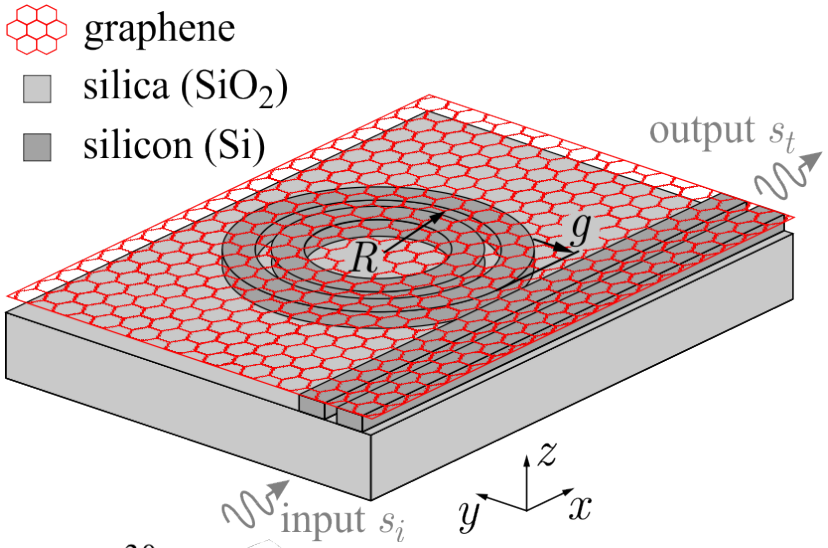
- GNR ring-resonator side-coupled to a similar bus w/g.
- GNR is formed on a uniform graphene sheet due to a biased uneven Si substrate.

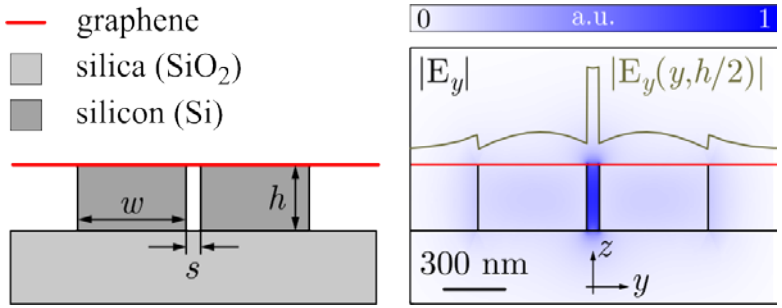




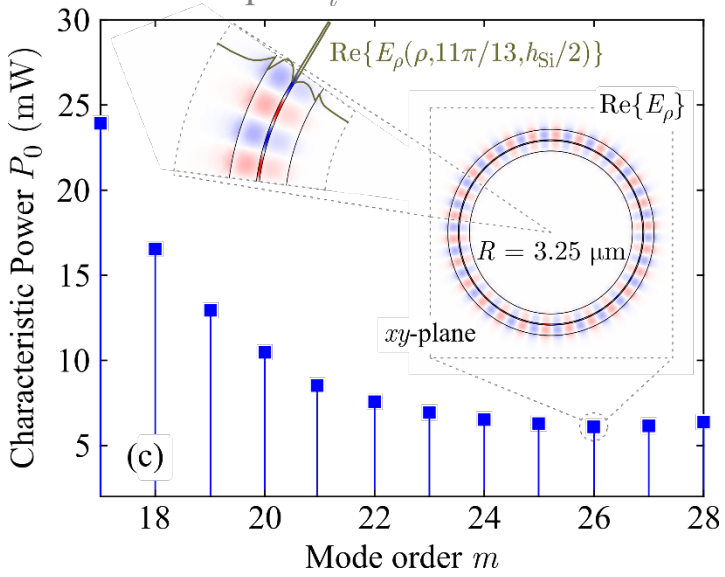
- Optimum mode at 10 THz has R = 10.1 um (m = 15) and characteristic power ~76 uW.
- Bistability with **theoretically infinite ER** is possible (at critical coupling) and at ~**400 uW input power**.

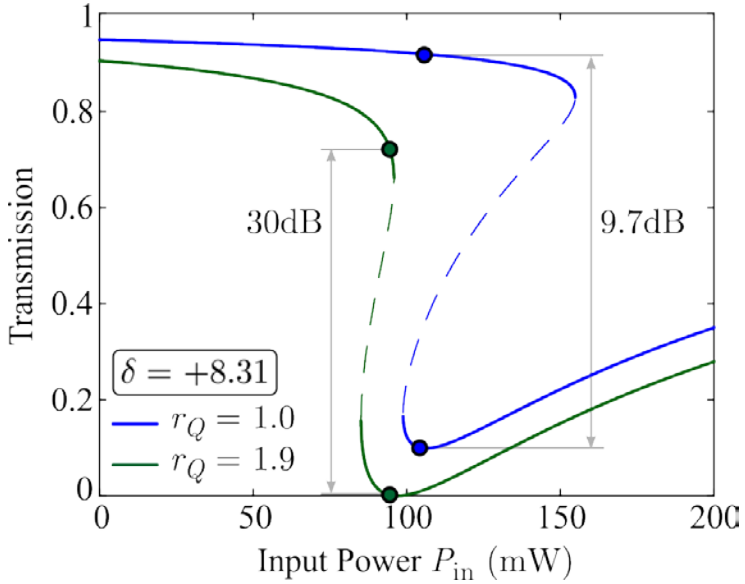
Optical Bistability / Silicon-slot TW resonator in NIR frequencies





- Si-slot w/g ring resonator side-coupled to a similar bus w/g.
- Si-slot w/g overlaid with graphene and TE mode profile.

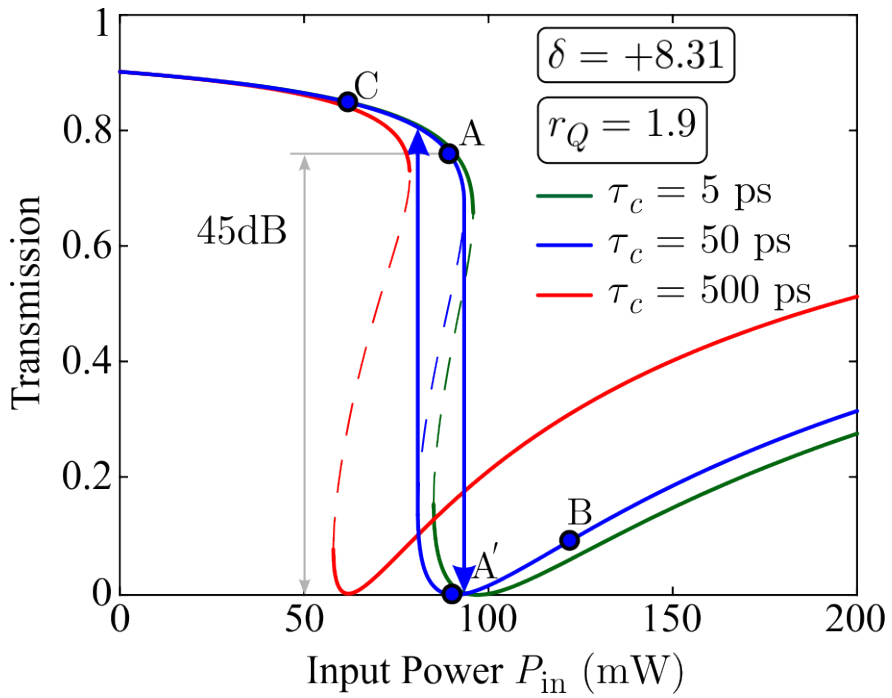




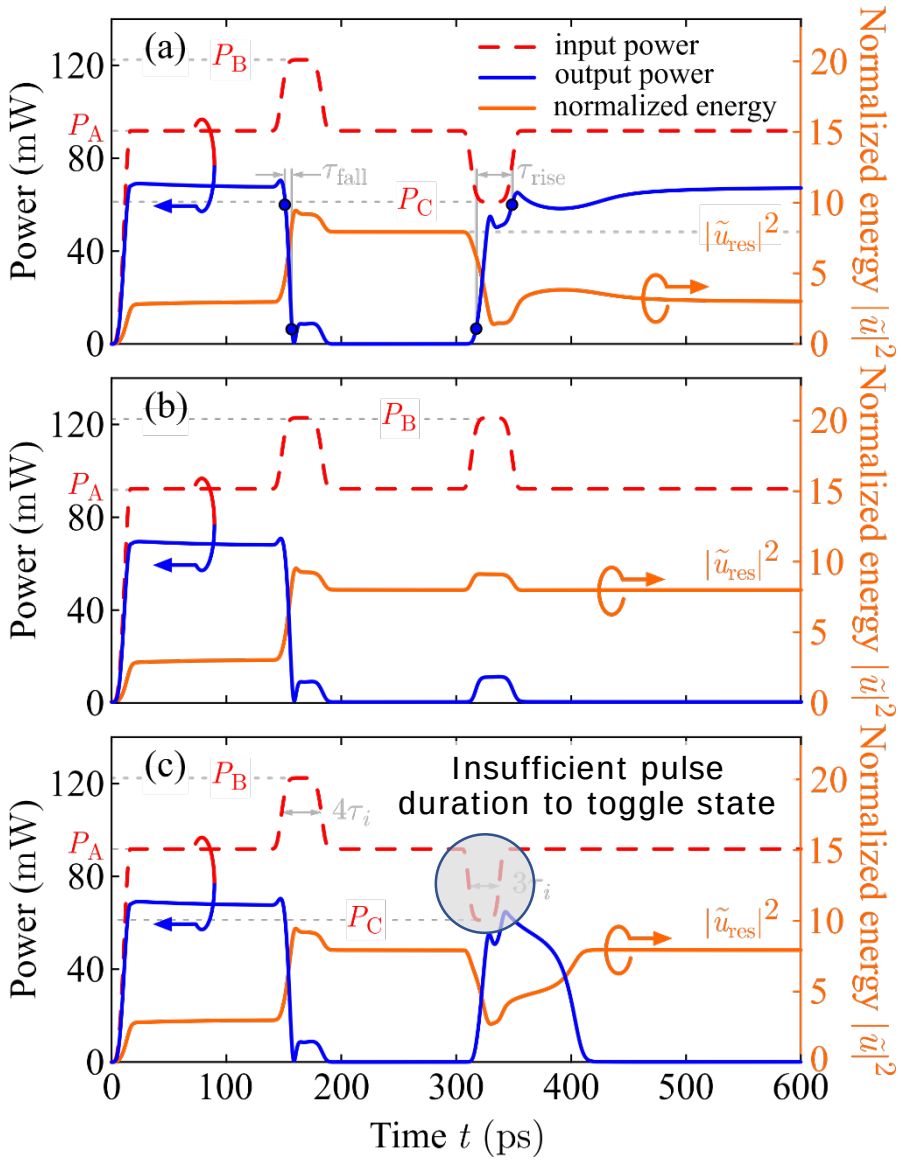
- Optimum mode has R = 3.25 um and characteristic power 6.1 mW.
- Bistability ignoring **free carrier effects (FCEs) in Si**. Maximum ER (inf) is for r_Q = 1.9 mostly due to **TPA in graphene**.

Optical Bistability / Silicon-slot TW resonator in NIR frequencies / Dynamics

- Bistability including **free carrier effects in Si** for different carrier lifetimes. For τ_c = 50 ps (through ion implantation or carrier sweeping), ER = 45 dB at P_{in} = 90 mW.



- **Memory operation**: Super Gaussian set & reset pulses accessing the route ABA'CA.



Christopoulos, *J. Appl. Phys.*, **122**, 233101,2017

Presentation Overview / At a glance

Optical properties of graphene: quick review

Linear surface conductivity
Nonlinear surface conductivity tensor

Optical Bistability

Bistability with TW resonators in the NIR
Bistability with TW resonators in the FIR

Degenerate Four Wave Mixing

FWM with SW resonators in the FIR

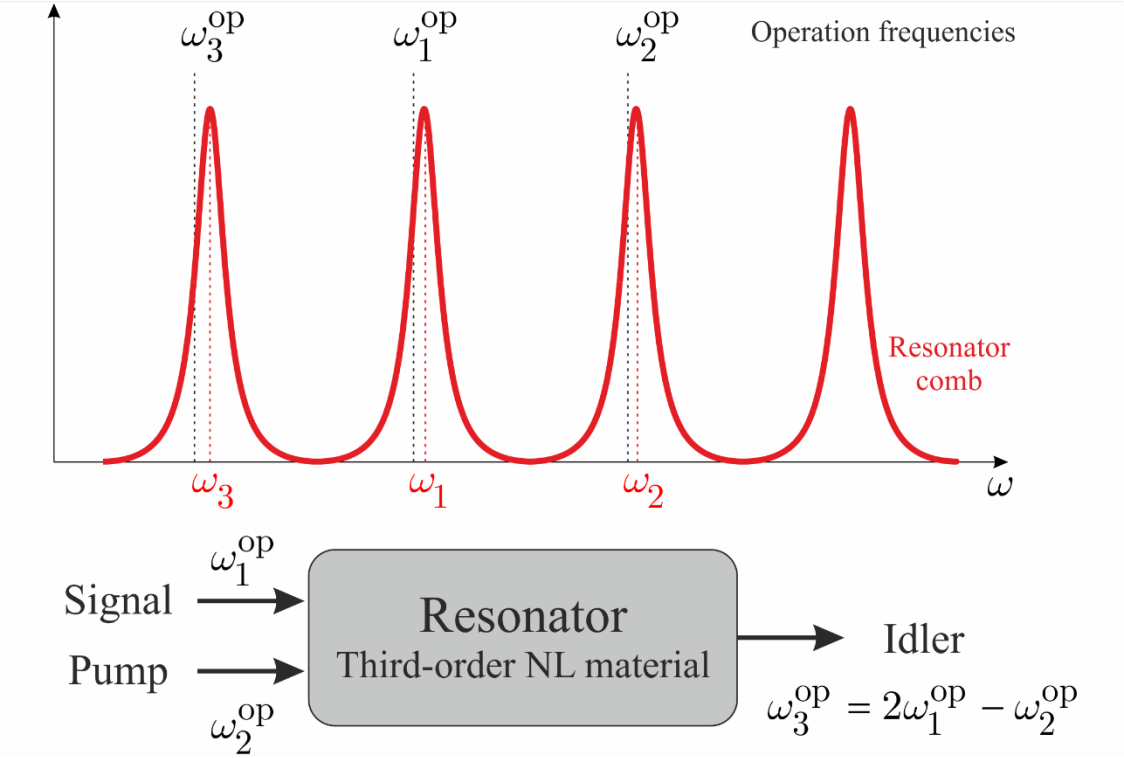
Saturable Absorption

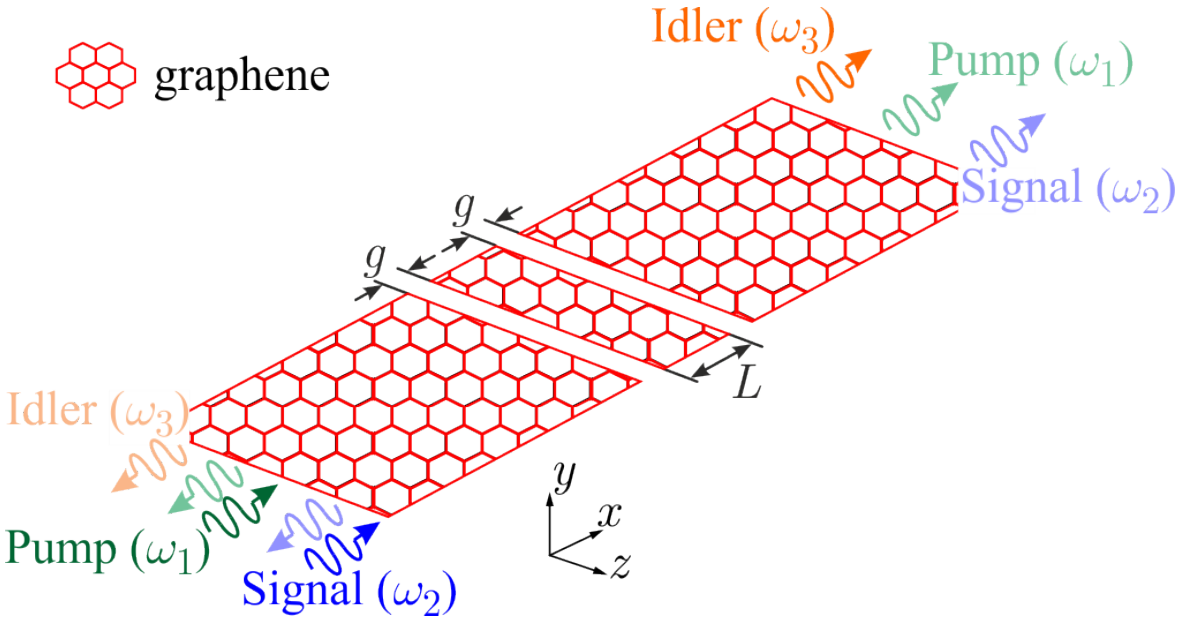
Intensity-dependent conductivity model
SA-induced switching/routing in nanophotonic disk resonators

Degenerate Four Wave Mixing / Basic concepts

Frequency generation and **wave mixing** in graphene resonators: multi-channel nonlinear interaction

- Automatic **phase matching**, intensity buildup leading to **high CE**, restrictions in operation frequencies
- **2D materials** → infinitesimally thin
- **Dispersion in linear properties** → graphene is highly dispersive at the FIR (THz)
- **SPM & XPM** present as well
- **CMT framework** and validation against **full-wave simulations**





Graphene plasmon-polariton standing-wave resonator, directly coupled to two semi-infinite graphene sheets

Degenerate Four Wave Mixing / Perturbation theory in graphene resonators

Nonlinear frequency shift (complex) with both polarization & current density nonlinearities. Allows modelling single-channel effects (SPM, Bistability, Self-Pulsation) and multi-channel (XPM, THG, FWM).

Nonlinear surface current

$$\frac{\Delta\omega}{\omega_0} = -\frac{\int \mathbf{P}_{\mathrm{NL}} \cdot \mathbf{E}_0^* \mathrm{d}^d r - \boxed{j\frac{1}{\omega_0}\int \mathbf{J}_{\mathrm{NL}} \cdot \mathbf{E}_0^* \mathrm{d}^{d-1} r}}{\int \varepsilon_0 \frac{\partial\{\omega\overline{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_0 \cdot \mathbf{E}_0^* \mathrm{d}^d r + \int \mu_0 \mathbf{H}_0 \cdot \mathbf{H}_0^* \mathrm{d}^d r + \int \frac{\partial\overline{\sigma}_{\mathrm{Im}}^{(1)}(\omega)}{\partial\omega}\mathbf{E}_0 \cdot \mathbf{E}_0^* \mathrm{d}^{d-1} r}$$

$$\left.\begin{aligned} \boldsymbol{\mathcal{J}}_{\mathrm{NL}} &= \overline{\sigma}^{(3)} \mid \boldsymbol{\mathcal{E}}\boldsymbol{\mathcal{E}}\boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{E}} &= \mathrm{Re}\{\mathbf{E}_1 \exp(j\omega_1 t) + \mathbf{E}_2 \exp(j\omega_2 t) + \mathbf{E}_3 \exp(j\omega_3 t)\} \end{aligned}\right\} \quad J_{\mathrm{NL},\mu}(\omega_k + \omega_\ell + \omega_m) = \frac{1}{4}\sum\nolimits_{\alpha\beta\gamma} \sigma_{\mu\alpha\beta\gamma}^{(3)} E_{k,\alpha} E_{\ell,\beta} E_{m,\gamma}$$

$$\mathbf{J}_{\mathrm{NL}}(\omega_k) = \mathbf{J}_{\mathrm{SPM}}(\omega_k) + \mathbf{J}_{\mathrm{XPM}}(\omega_k) + \mathbf{J}_{\mathrm{DFWM}}(\omega_k)$$

SPM $\quad \omega_k = \omega_k - \omega_k + \omega_k$

XPM $\quad \omega_k = \omega_k - \omega_\ell + \omega_\ell = \omega_k - \omega_m + \omega_m$

FWM
$$\left.\begin{aligned} \omega_1 &= -\omega_1 + \omega_2 + \omega_3, \\ \omega_2 &= 2\omega_1 - \omega_3, \\ \omega_3 &= 2\omega_1 - \omega_2 \end{aligned}\right\}$$

$$\mathbf{J}_{\mathrm{DFWM}}(\omega_1) = \tfrac{1}{4}\sigma_3 \left[2(\mathbf{E}_{2,\parallel} \cdot \mathbf{E}_{3,\parallel})\mathbf{E}_{1,\parallel}^* + 2(\mathbf{E}_{1,\parallel}^* \cdot \mathbf{E}_{3,\parallel})\mathbf{E}_{2,\parallel} + 2(\mathbf{E}_{1,\parallel}^* \cdot \mathbf{E}_{2,\parallel})\mathbf{E}_{3,\parallel}\right]$$

$$\mathbf{J}_{\mathrm{DFWM}}(\omega_2) = \tfrac{1}{4}\sigma_3 \left[2(\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{3,\parallel}^*)\mathbf{E}_{1,\parallel} + (\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{1,\parallel})\mathbf{E}_{3,\parallel}^*\right]$$

$$\mathbf{J}_{\mathrm{DFWM}}(\omega_3) = \tfrac{1}{4}\sigma_3 \left[2(\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{2,\parallel}^*)\mathbf{E}_{1,\parallel} + (\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{1,\parallel})\mathbf{E}_{2,\parallel}^*\right]$$

Degenerate Four Wave Mixing / Perturbation theory in graphene resonators

Full nonlinear frequency shift experienced by the three resonance modes $\omega_1, \omega_2, \omega_3$:

$$\Delta\omega_1 a_1 = -\gamma_{11}|a_1|^2 a_1 - 2\gamma_{12}|a_2|^2 a_1 - 2\gamma_{13}|a_3|^2 a_1 - 2\beta_1 a_1^* a_2 a_3$$
$$\Delta\omega_2 a_2 = -\gamma_{22}|a_2|^2 a_2 - 2\gamma_{21}|a_1|^2 a_2 - 2\gamma_{23}|a_3|^2 a_2 - \beta_2 a_1^2 a_3^*$$
$$\Delta\omega_3 a_3 = -\gamma_{33}|a_3|^2 a_3 - 2\gamma_{31}|a_1|^2 a_3 - 2\gamma_{32}|a_2|^2 a_3 - \beta_3 a_1^2 a_2^*$$

Nonlinear coefficients including bulk & surface contributions

$\gamma_{k\ell}$ SPM/XPM

β_k FWM

Christopoulos, *Phys. Rev. B*, **98**, 235421,2018

$$\beta_k = 4\left(\frac{\omega_k}{c_0}\right)^d \omega_k c_0 \kappa_{k,b}^{\mathrm{DFWM}} n_2^{\max} + \left(\frac{\omega_k}{c_0}\right)^{d+1} \kappa_{k,s}^{\mathrm{DFWM}} \frac{\sigma_{3,\mathrm{Im}}^{\max}}{\varepsilon_0^2}$$

$$\kappa_{1,b}^{\mathrm{DFWM}} = \left(\frac{c_0}{\omega_1}\right)^d \frac{\frac{1}{3}\int n^2 n_2 \left[2(\mathbf{E}_1^* \cdot \mathbf{E}_3)(\mathbf{E}_1^* \cdot \mathbf{E}_2) + (\mathbf{E}_1^* \cdot \mathbf{E}_1^*)(\mathbf{E}_2 \cdot \mathbf{E}_3)\right] \mathrm{d}^d r}{\frac{16}{\varepsilon_0^2} W_{\mathrm{res},1} W_{\mathrm{res},2}^{1/2} W_{\mathrm{res},3}^{1/2} n_2^{\max}}$$

Bulk nonlinear feedback parameter

$$\kappa_{1,s}^{\mathrm{DFWM}} = \left(\frac{c_0}{\omega_1}\right)^{d+1} \frac{\int \sigma_{3,\mathrm{Im}} \left[2(\mathbf{E}_{1,\parallel}^* \cdot \mathbf{E}_{3,\parallel})(\mathbf{E}_{1,\parallel}^* \cdot \mathbf{E}_{2,\parallel}) + (\mathbf{E}_{1,\parallel}^* \cdot \mathbf{E}_{1,\parallel}^*)(\mathbf{E}_{2,\parallel} \cdot \mathbf{E}_{3,\parallel})\right] \mathrm{d}^{d-1} r}{\frac{16}{\varepsilon_0^2} W_{\mathrm{res},1} W_{\mathrm{res},2}^{1/2} W_{\mathrm{res},3}^{1/2} \sigma_{3,\mathrm{Im}}^{\max}}$$

Surface nonlinear feedback parameter

Symmetries in DFWM nonlinear coefficients

$$\frac{\beta_{1,b}}{\omega_1} = \frac{\beta_{2,b}^*}{\omega_2} = \frac{\beta_{3,b}^*}{\omega_3}$$
$$\beta_{1,s} = \beta_{2,s}^* = \beta_{3,s}^*$$

Degenerate Four Wave Mixing / Coupled Mode Theory

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = j\left(\omega_1 - \gamma_{11}|a_1|^2 - 2\gamma_{12}|a_2|^2 - 2\gamma_{13}|a_3|^2\right)a_1 - j2\beta_1 a_1^* a_2 a_3 - \left(\frac{1}{\tau_{i,1}} + \frac{1}{\tau_{e,1}}\right)a_1 + \sqrt{\frac{1}{\tau_{e,1}}}s_{\mathrm{in},1},$$

$$\frac{\mathrm{d}a_2}{\mathrm{d}t} = j\left(\omega_2 - 2\gamma_{21}|a_1|^2 - \gamma_{22}|a_2|^2 - 2\gamma_{23}|a_3|^2\right)a_2 - j\beta_2 a_1^2 a_3^* - \left(\frac{1}{\tau_{i,2}} + \frac{1}{\tau_{e,2}}\right)a_2 + \sqrt{\frac{1}{\tau_{e,2}}}s_{\mathrm{in},2},$$

$$\frac{\mathrm{d}a_3}{\mathrm{d}t} = j\left(\omega_3 - 2\gamma_{31}|a_1|^2 - 2\gamma_{32}|a_3|^2 - \gamma_{33}|a_3|^2\right)a_3 - j\beta_3 a_1^2 a_2^* - \left(\frac{1}{\tau_{i,3}} + \frac{1}{\tau_{e,3}}\right)a_3 + \sqrt{\frac{1}{\tau_{e,3}}}s_{\mathrm{in},3},$$

Self-phase modulation (SPM)

Cross-phase modulation (XPM)

Degenerate four-wave-mixing (DFWM)

Intrinsic and external cavity loss

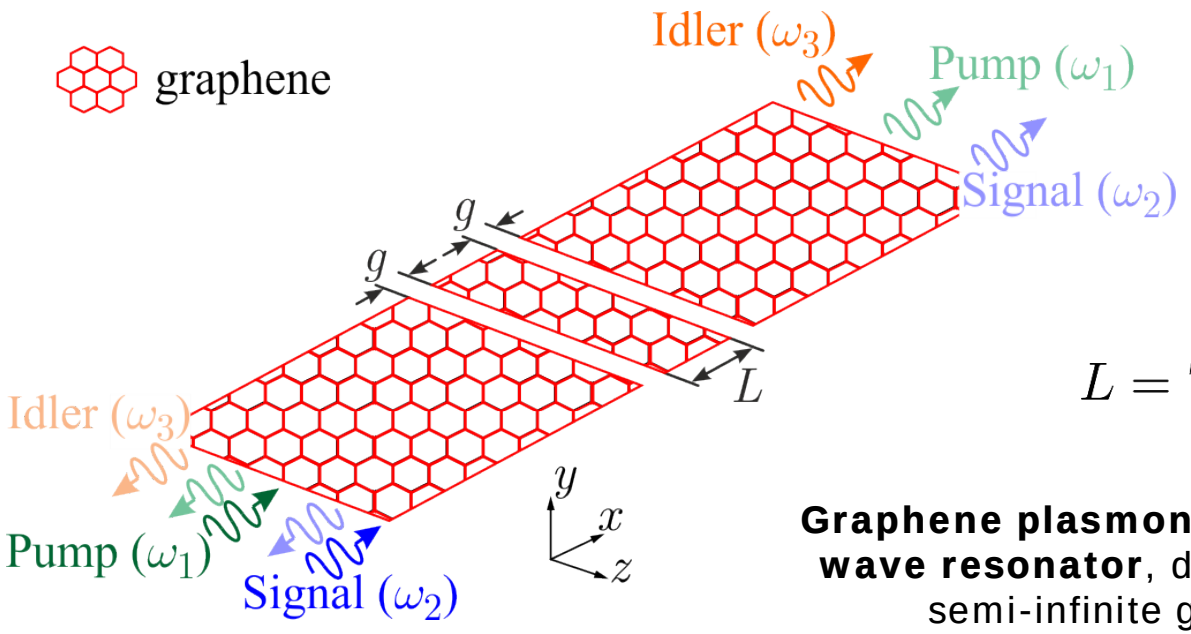
Input waves & Output waves

$$s_{\mathrm{out},k} = \sqrt{\frac{1}{\tau_{e,k}}}a_k$$

- Can be cast in a normalized form introducing a characteristic power level and the frequency detuning with respect to the cavity resonances.

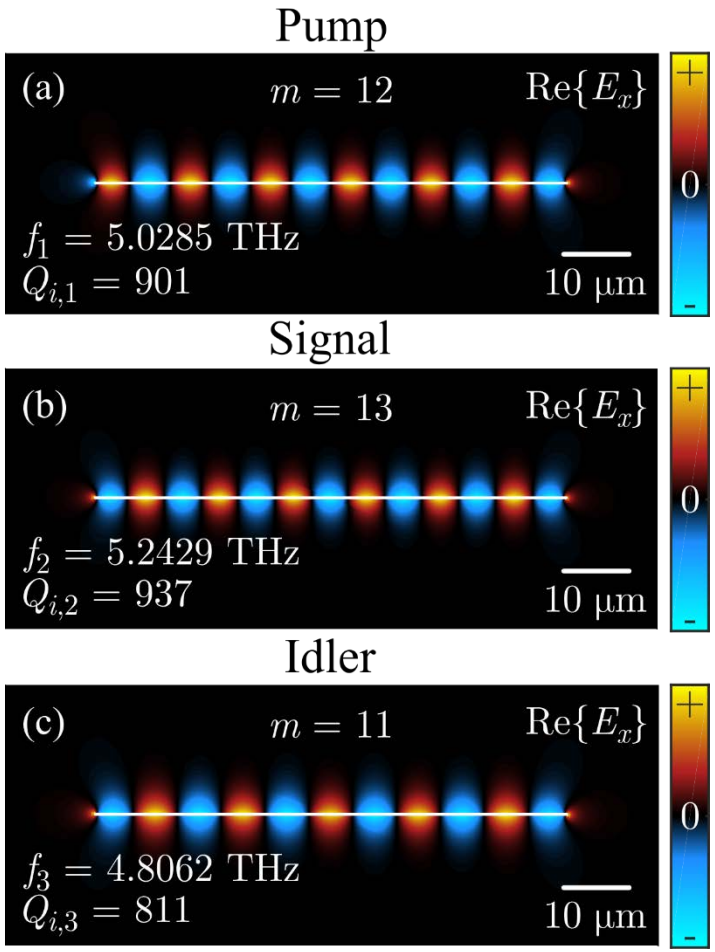
Christopoulos, *Phys. Rev. B*, **98**, 235421,2018

Degenerate Four Wave Mixing / Standing-wave resonator system for FIR frequencies



$$L = 72\ \mu\text{m}$$

Graphene plasmon-polariton standing-wave resonator, directly coupled to two semi-infinite graphene sheets



$$g = 1\ \mu\text{m} \qquad Q_{e,1} = 326, Q_{e,2} = 386, Q_{e,3} = 267$$

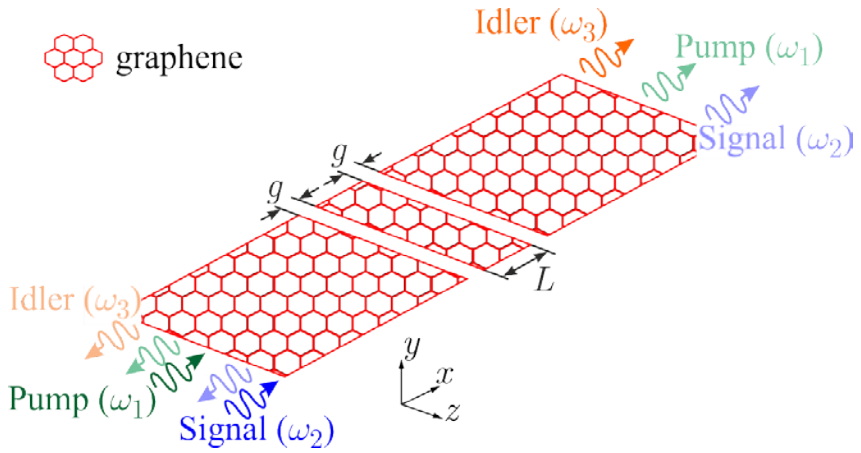
- Calculation of the Q-factors (intrinsic & external) should be done with extra care, see:

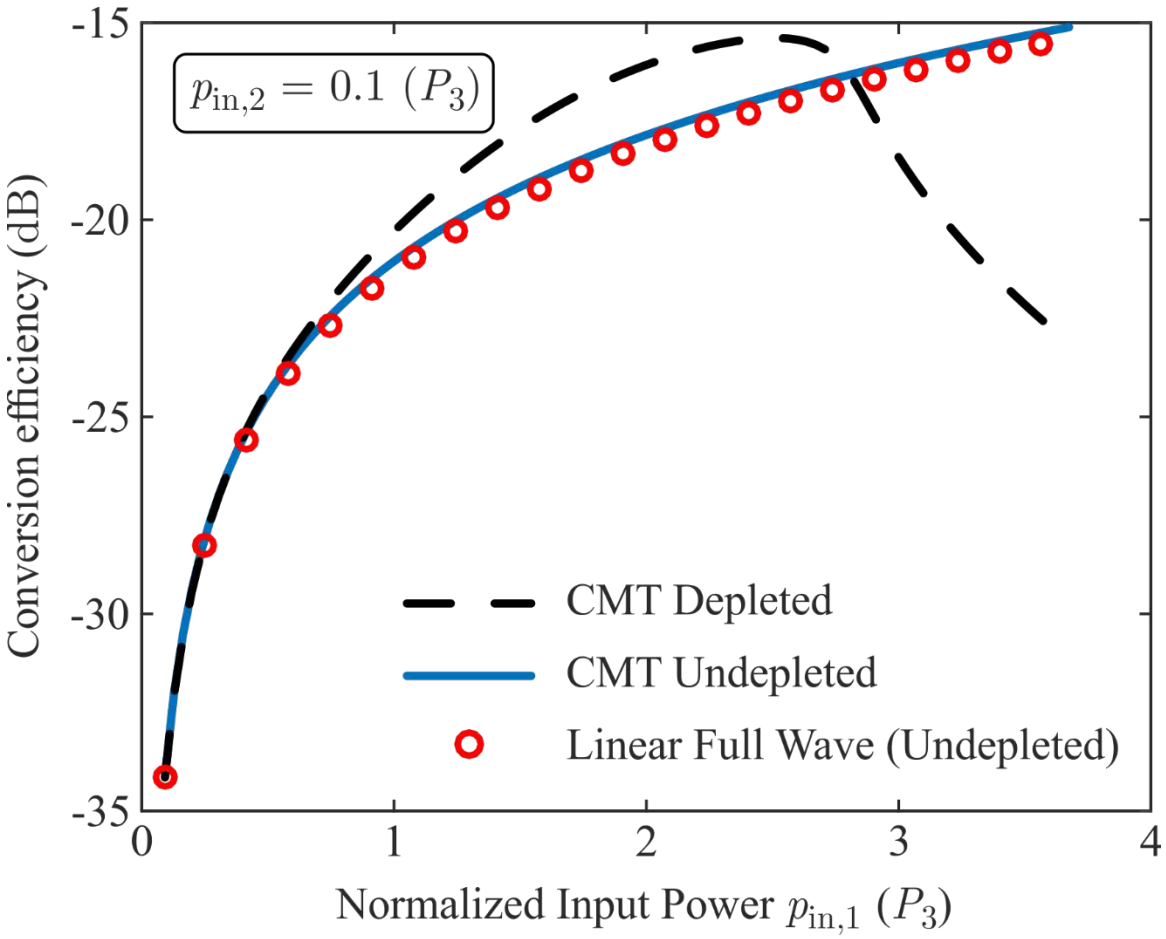
  Christopoulos, *Opt. Express*, **27**, 14505,2019

Due to material (or waveguide) dispersion

$$\omega_3 \neq 2\omega_1 - \omega_2$$

Degenerate Four Wave Mixing / Validation of CMT framework against FEM simulations





Full-wave linear simulations (three independent):

Step #1: Linear simulations at $\omega_1^{\text{op}} = \omega_1, \omega_2^{\text{op}} = \omega_2$

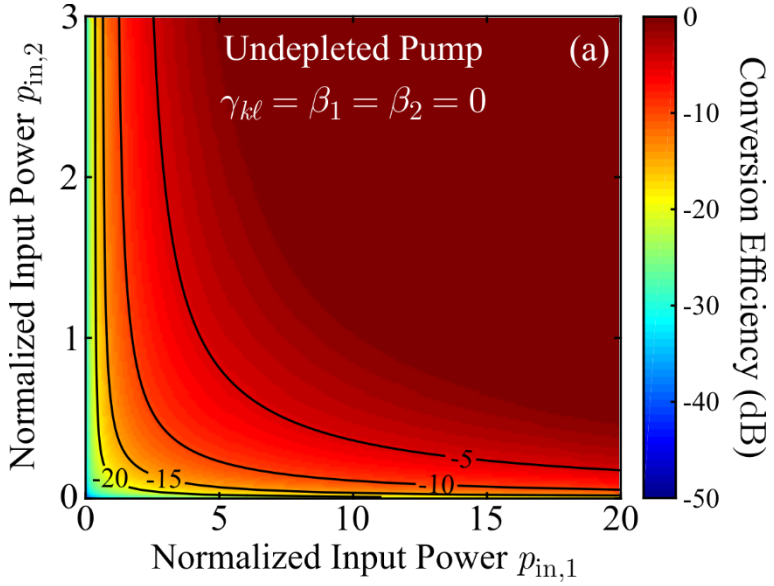
Step #2: Compute current at $\omega_3^{\text{op}} = 2\omega_1^{\text{op}} - \omega_2^{\text{op}}$, using

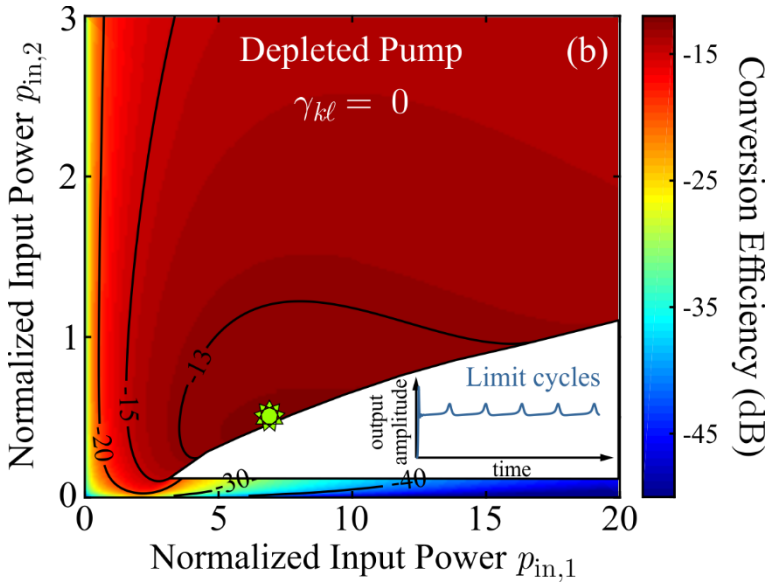
$$\mathbf{J}(\omega_3^{\text{op}}) = \frac{1}{4}\sigma_3 \left[2(\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{2,\parallel}^*)\mathbf{E}_{1,\parallel} + (\mathbf{E}_{1,\parallel} \cdot \mathbf{E}_{1,\parallel})\mathbf{E}_{2,\parallel}^*\right]$$

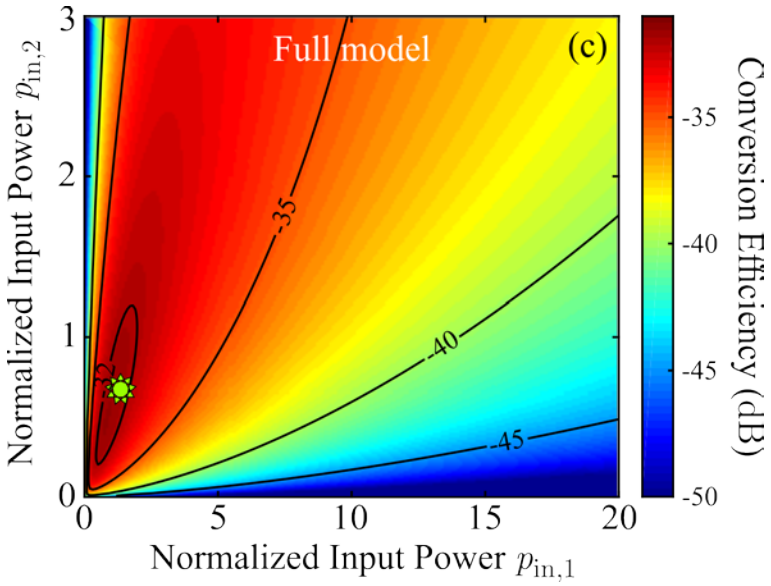
Step #3: Use Step #2 current as source to calculate E-field at ω_3^{op}

- Power **lost from pump or signal** is **not accounted** and nonlinear interactions between pump-idler and signal-idler **are lost**.
- This is the **undepleted pump** case and is **accurate for low / medium conversion efficiencies**.

Degenerate Four Wave Mixing / Conversion Efficiency (CE)







- **Undepleted pump case**:
  CE monotonically increases (unrealistic)

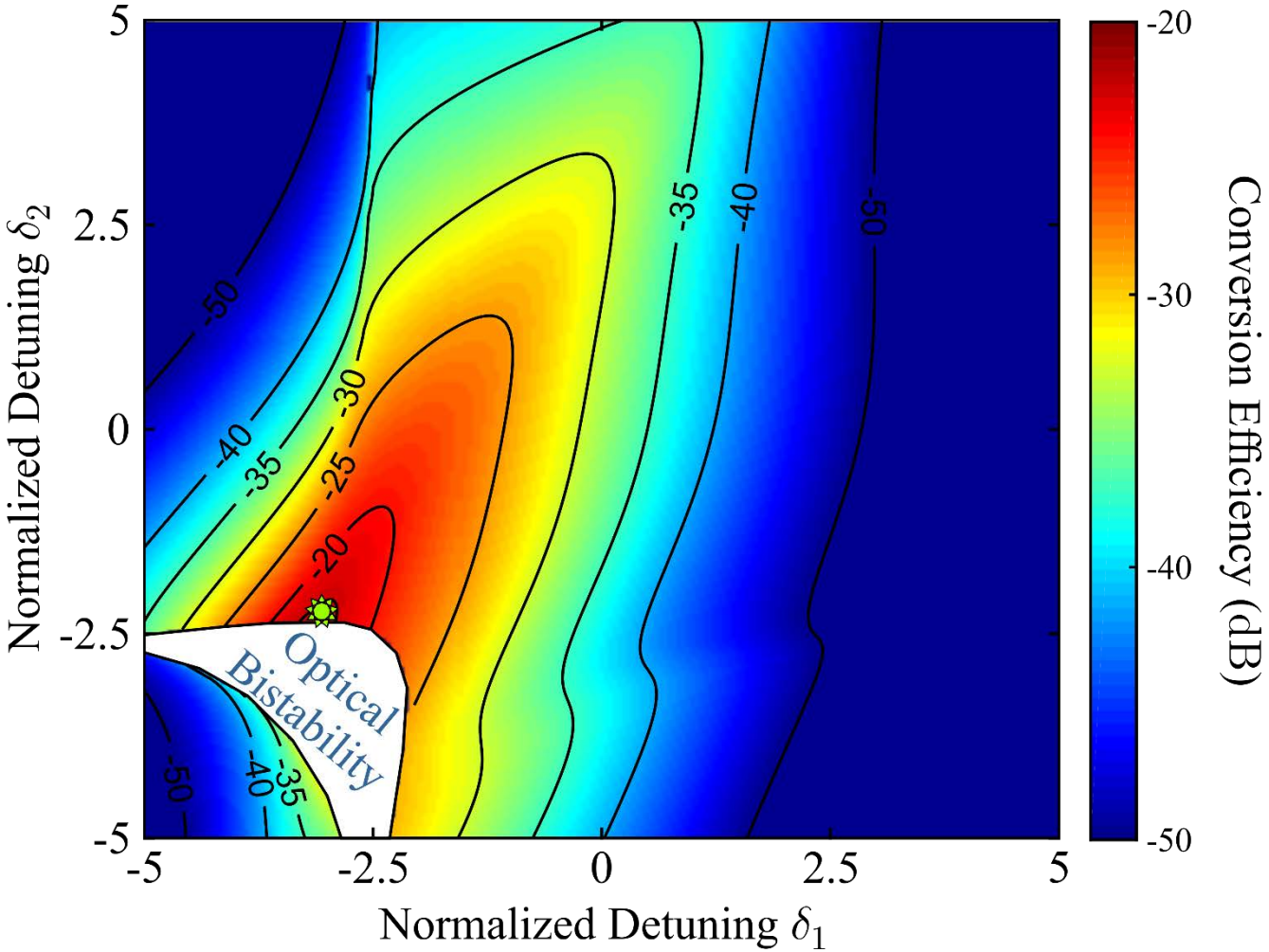
- **Depleted pump case**:
  max{CE}=-12.5 dB
  at $p_{in,1}$=7.0 & $p_{in,2}$ = 0.5
  (at the boundary of unstable operation)

- **Full model** (includes SPM/XPM):
  max{CE}=-31.5 dB
  at $p_{in,1}$=1.0 & $p_{in,2}$ = 0.6

❖ **Reason for decreased CE:** SPM & XPM shift the cavity resonances away from the operating frequencies, resulting in nonzero detuning and suboptimal light-cavity coupling.

❖ **Solution:** Pre-shift operating frequencies w.r.t. the unperturbed resonance frequencies to accommodate the red-shift introduced by the Kerr effect (SPM/XPM).

Degenerate Four Wave Mixing / Conversion Efficiency (CE)



Conversion Efficiency (full-model) at $p_{in,1}$=1.0 & $p_{in,2}$ = 0.6

- Pre-shifting **greatly improves CE**. At δ_1=-3.1 and δ_2=-2.35 **CE = -17.6 dB** from -31.5 dB. Further fine tuning is possible w.r.t. input power.

- Regions of **optical bistability** are also predicted, though unfavorable for FWM.

$$\delta_k < -(1 + r_{Q,k})\sqrt{3}$$

Christopoulos, *Phys. Rev. B*, **98**, 235421,2018

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Bistability with TW resonators in the FIR

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FWM with SW resonators in the FIR

Saturable Absorption

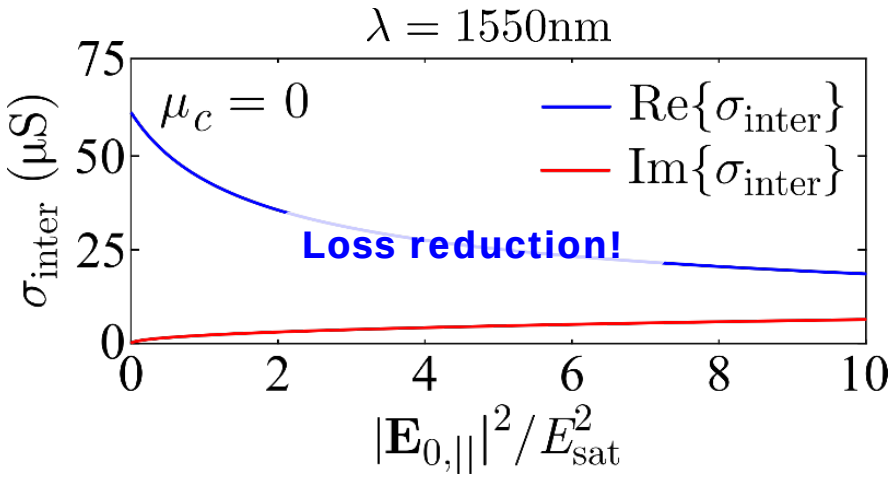
Intensity-dependent conductivity model
SA-induced switching/routing in nanophotonic disk resonators

Saturable absorption / Intensity-dependent conductivity model

$$\sigma_c = \sigma_{\text{intra}} + \sigma_{\text{inter}}(I) \longrightarrow \sigma_{\text{inter}} = \sigma_0 \left[\frac{1}{\sqrt{1+3|\mathbf{E}_{0,\parallel}|^2/E_{\text{sat}}^2}} + j\frac{1-\exp\{-\eta_1\sqrt{|\mathbf{E}_{0,\parallel}|^2/E_{\text{sat}}^2}\}}{\sqrt{1+(\eta_1^2|\mathbf{E}_{0,\parallel}|^2/E_{\text{sat}}^2)^{0.4}}} \right]$$

Marini, *Phys. Rev. Lett.* **116**, 125408, 2016
Marini, *Phys. Rev. B.* **95**, 217401, 2017

- **Intraband conductivity** does not saturate
- **Interband conductivity** saturates and both Re/Im parts are affected
- **Saturation intensity** can be very low $I_{\text{sat}} = E_{\text{sat}}^2/2\eta_0 \sim 1\ \text{MW/cm}^2$

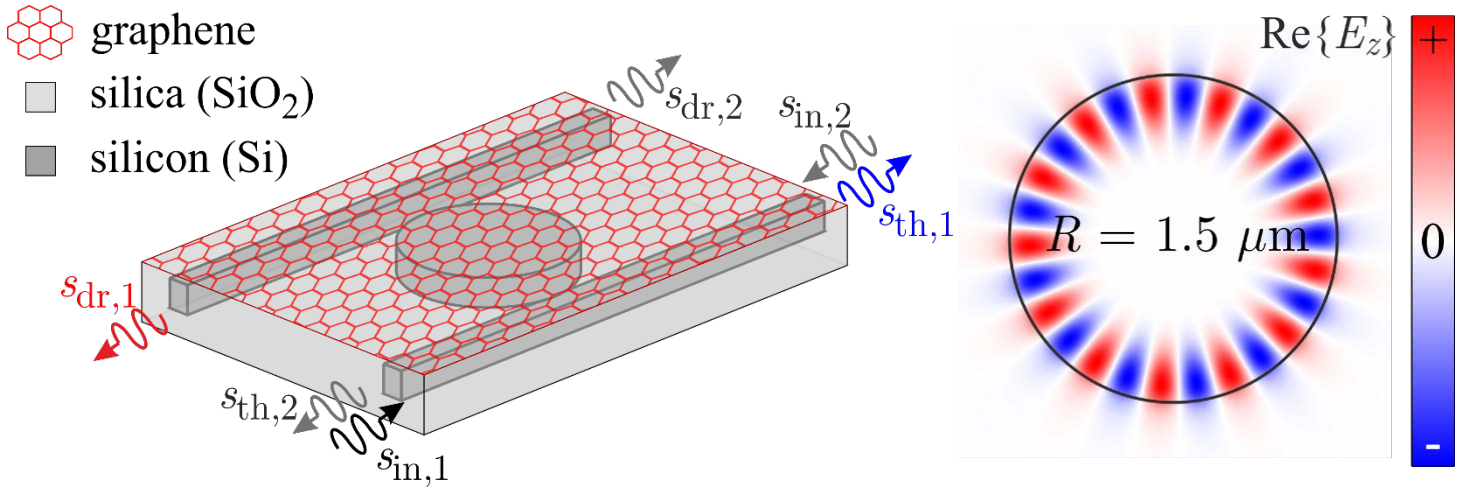


Coupled Mode Theory including SA (here excluding SPM/TPA/FCEs):

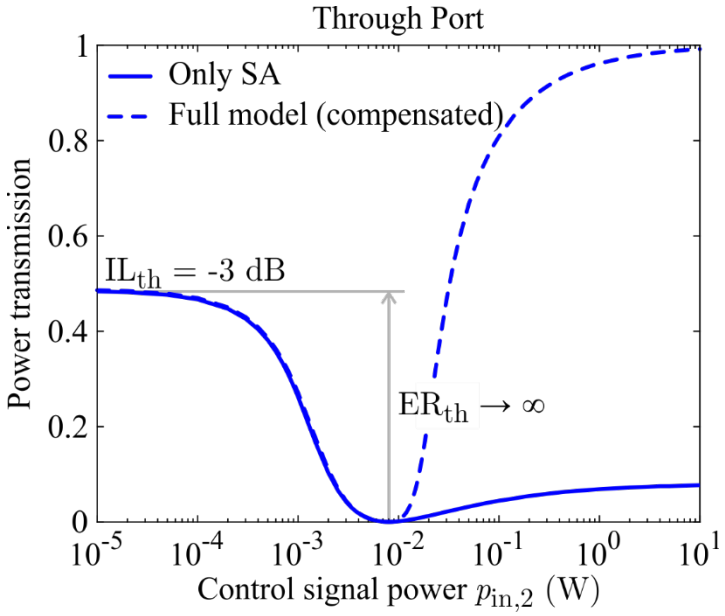
$$\frac{\mathrm{d}a}{\mathrm{d}t} = j(\omega_0 + \Delta\omega_{\text{intra}} + \Delta\omega_{\text{inter}})a - \left(\frac{1}{\tau_{\text{SA}}} + \frac{1}{\tau_{\text{intra}}} + \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_e}\right)a + j\sqrt{\frac{2}{\tau_e}}s_{\text{in}}$$

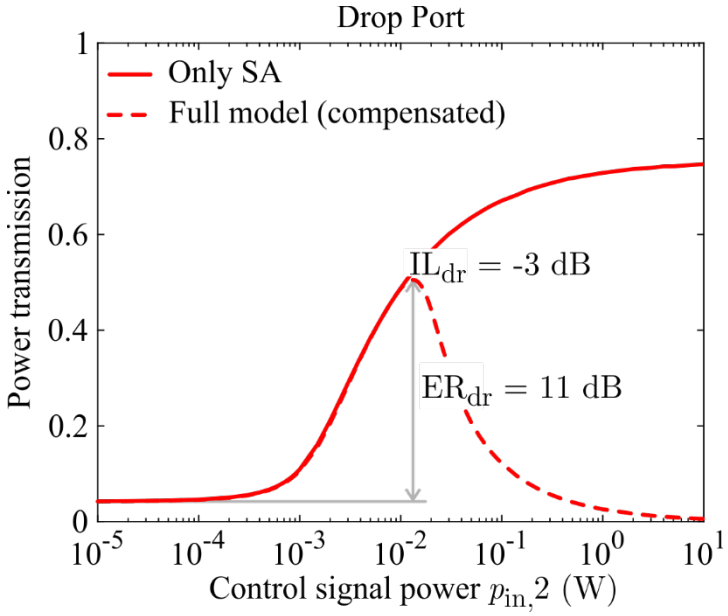
$$\Delta\omega_{\text{inter}} = -\frac{1}{4}\int \sigma_0 \frac{1-\exp\{-\eta_1\sqrt{|a|^2|\mathbf{E}_{\text{ref},\parallel}|^2/E_{\text{sat}}^2}\}}{\sqrt{1+(\eta_1^2|a|^2|\mathbf{E}_{\text{ref},\parallel}|^2/E_{\text{sat}}^2)^{0.4}}}|\mathbf{E}_{\text{ref},\parallel}|^2\mathrm{d}^{d-1}r \qquad \frac{1}{\tau_{\text{SA}}} = +\frac{1}{4}\int \frac{\sigma_0}{\sqrt{1+3|a|^2|\mathbf{E}_{\text{ref},\parallel}|^2/E_{\text{sat}}^2}}|\mathbf{E}_{\text{ref},\parallel}|^2\mathrm{d}^{d-1}r$$

Saturable absorption / Ultrafast nanophotonic switching-routing element in the NIR



- Silica-clad silicon disk coupled to Si-wires, with uniform graphene layer on-top. Graphene introduces **SA**, linear losses and **nonlinear frequency shift** (Kerr).
- **Wavelength #1** is **controlled** by the **high-power wavelength #2**. Output can be directed to the "through" or "drop" ports.



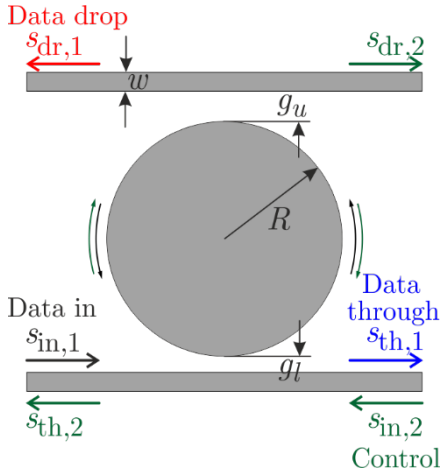


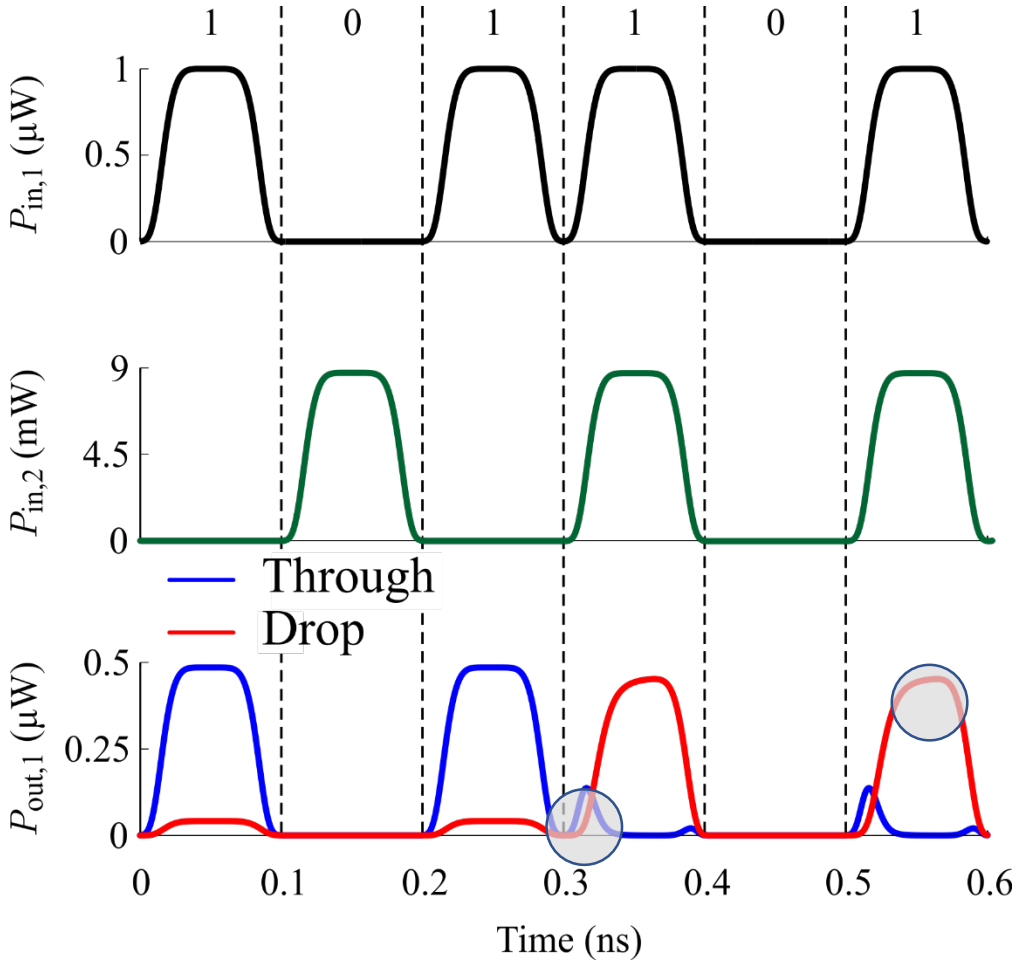
- **Through Port**: Control wavelength provides critical coupling and theoretically allows for infinite ER.
- **Drop Port**: ER is finite but sufficiently high.
- **Control power** is **~9 mW** and Kerr nonlinearities become important > 10 mW.

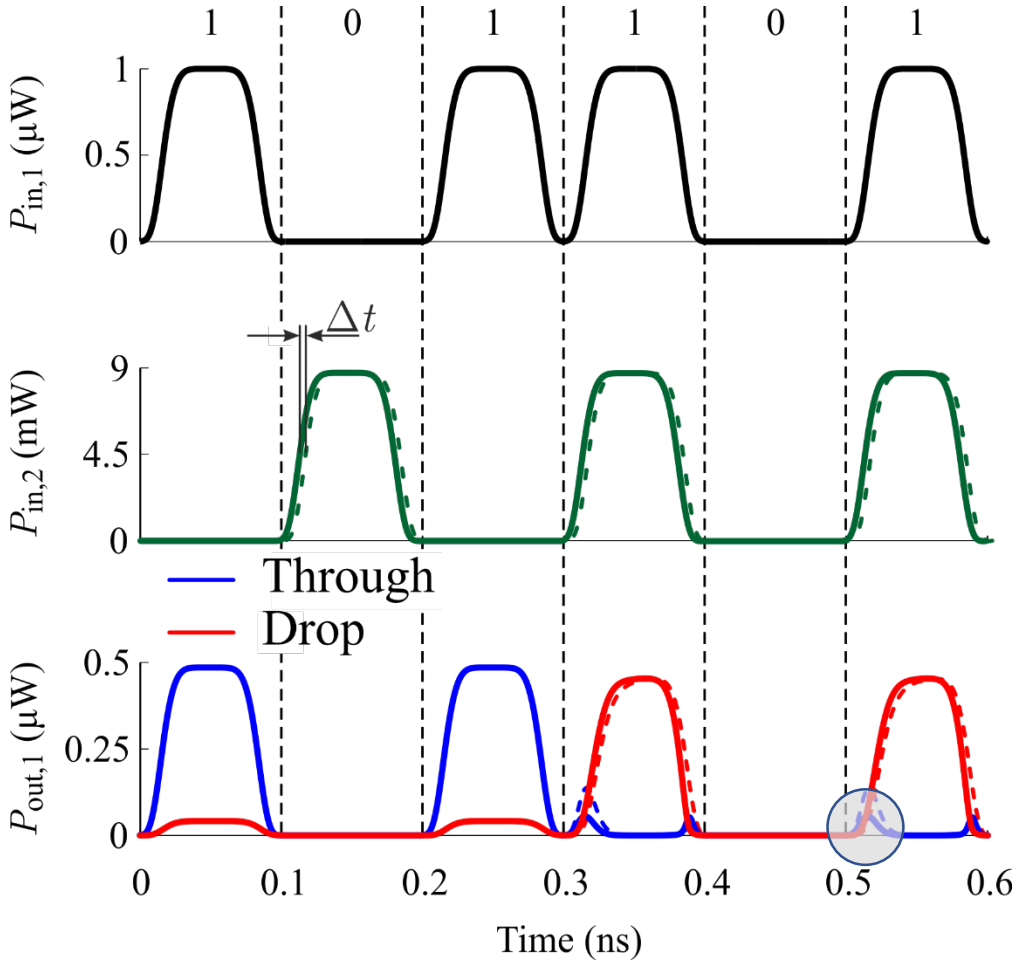
Ataloglou, *Phys. Rev. A.* **97**, 063836, 2018

Ataloglou, *EOSAM 2018* (Delft)

Saturable absorption / Ultrafast nanophotonic switching-routing element in the NIR







- **IL** of -3 dB, 10 dB **ER** at drop port, 10-50 Gbps **rate**, some pulse shape **distortion**.
- **Time offset** in control helps **early cavity filling** and suppresses distortions.

Ataloglou, *EOSAM 2018* (Delft)

Conclusions

- **Single-channel nonlinearities / Bistability:**

  Can be exploited both in the photonic (NIR) or plasmonic (FIR) layouts.

  High Extinction Ratios (ER) and low input power levels are possible.

  ~400 uW at FIR (10 THz), ~90 mW at NIR (1550 nm).

- **Degenerate Four Wave Mixing:**

  High Conversion Efficiencies (CE) at the FIR attributed to the tightly-confined graphene plasmons and the high nonlinear surface conductivity.

  CE ~ -17 dB, with further prospect for improvement.

- **Saturable Absorption:**

  Saturation intensities can be very low and thus SA can become the dominant NL effect.

  Optically-controlled switching/routing elements are possible in the NIR at ~10 mW control power.

- **Coupled Mode Theory (CMT) framework:**

  Highly accurate and flexible – allows systematic incorporation of a broad range of single or multi-channel NL effects. Excellent agreement against FEM simulations for CW conditions.

Co-authors

ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ

Mr. Thomas Christopoulos

FORTH
INSTITUTE OF ELECTRONIC STRUCTURE AND LASER

Dr. Odysseas Tsilipakos

ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ

Mr. Georgios Sinatkas

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Mr. Vasileios Ataloglou

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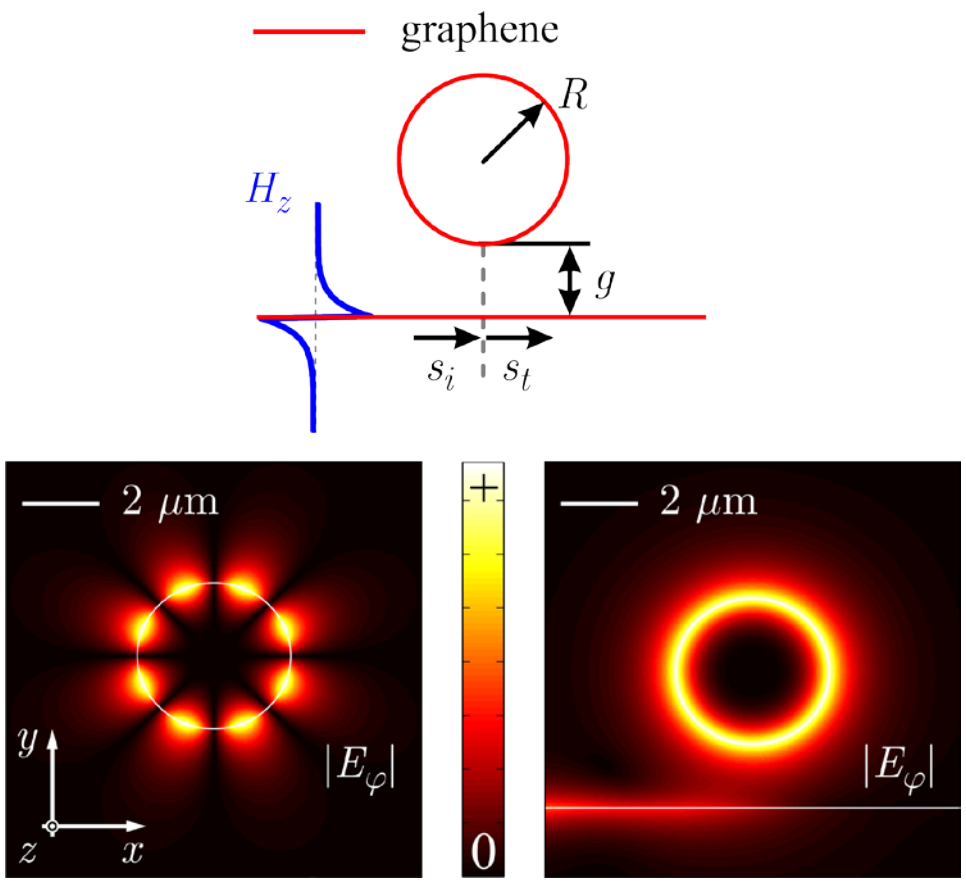
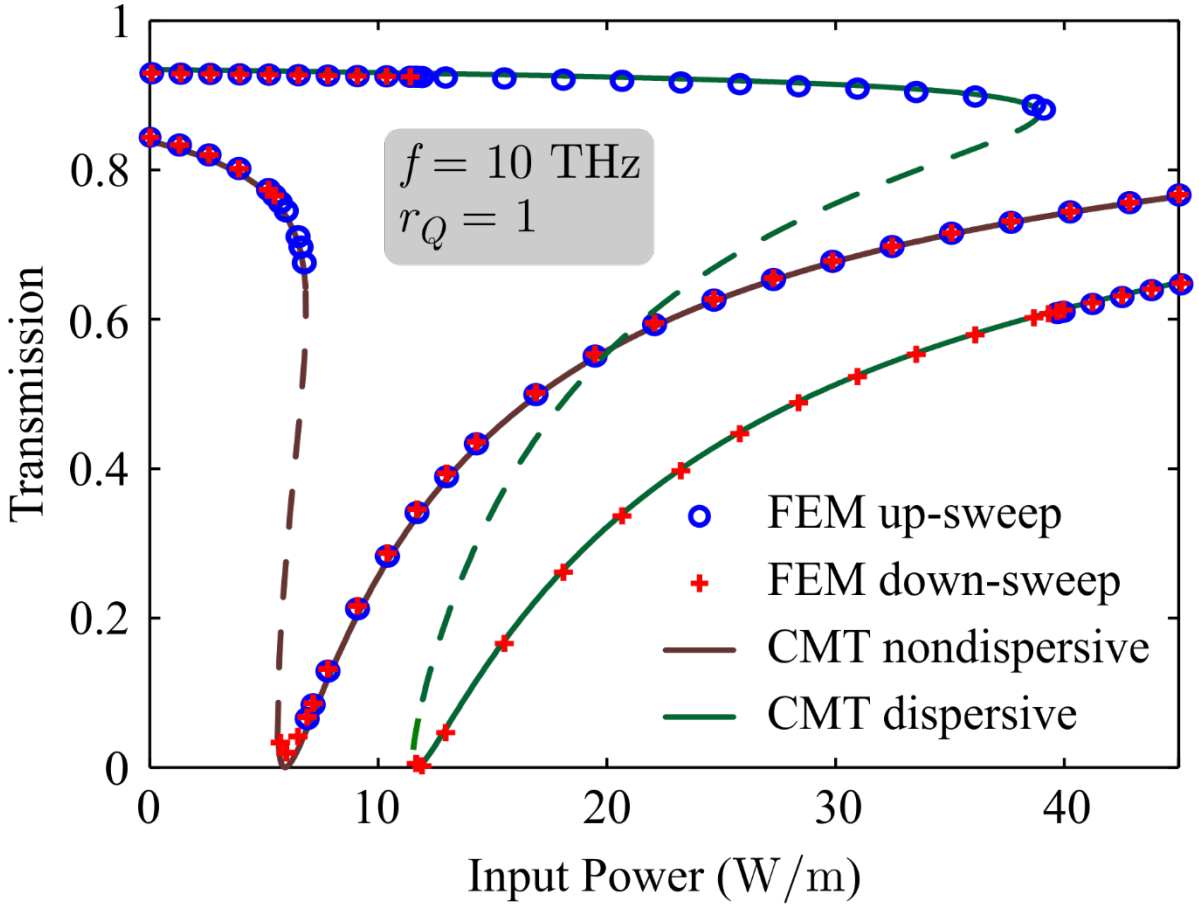
Backup slides / Validation of CMT bistability curves against FEM simulations





- Infinite graphene tube (TW resonator) side-coupled to an infinite graphene sheet (w/g): eigenvalue problem (10 THz) and E-field distribution when feeding the bus w/g.
- Bistability curves (dispersive & nondispersive case) obtained with CMT and full-wave nonlinear FEM simulations.